GrEnFIn

Greening Energy
Market and Finance

University of Economics in Katowice

The transition risk and the regulations: Green bonds and beyond

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Schedule

1. **Green Bond characteristics**
2. **Green Bond market**
3. **Bond's price and yield relation**
4. **Coupon calculation of a new issue**
5. **Green bond yield curve**

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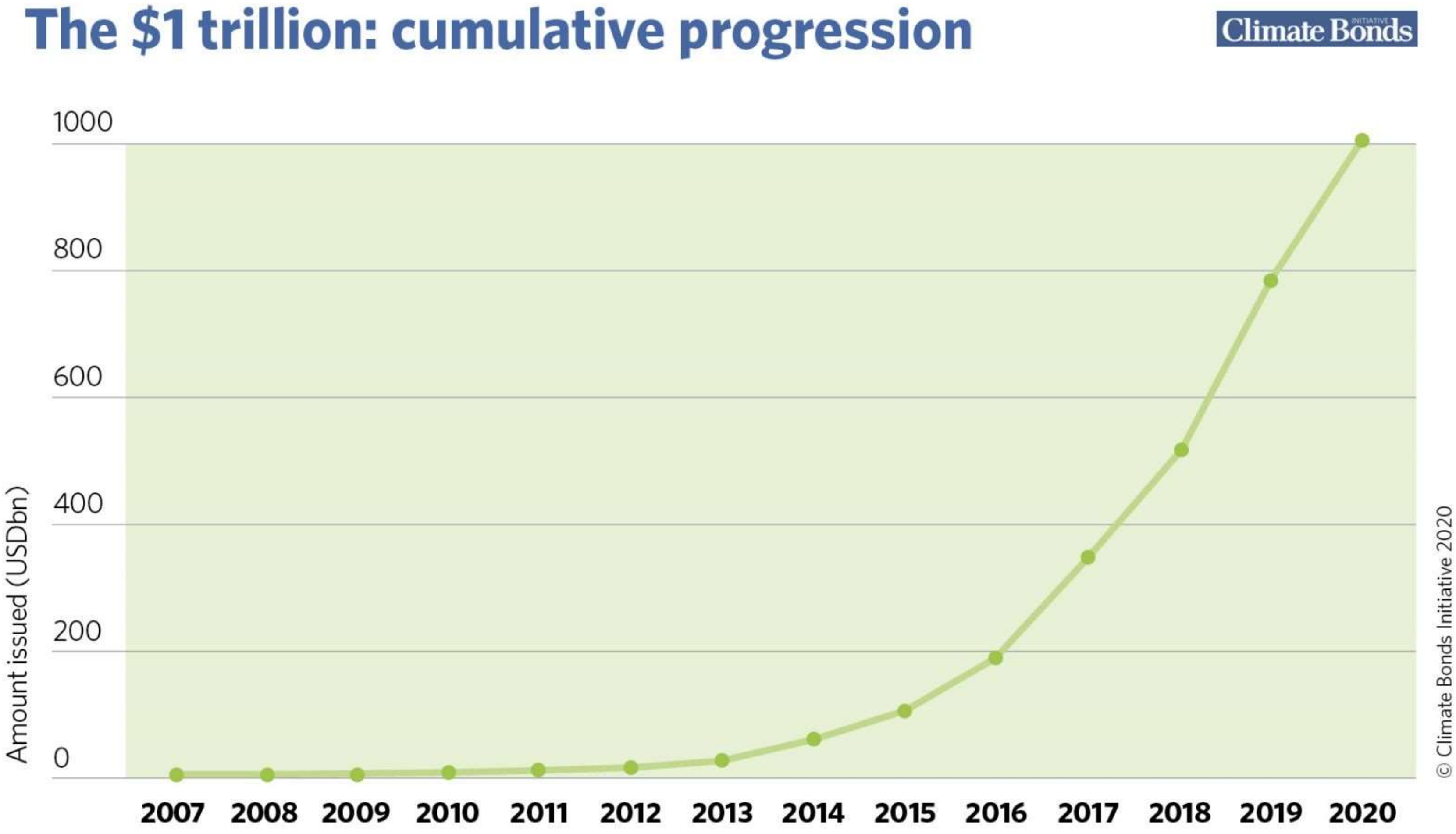
GREEN BOND CHARACTERSISTICS

Green bond



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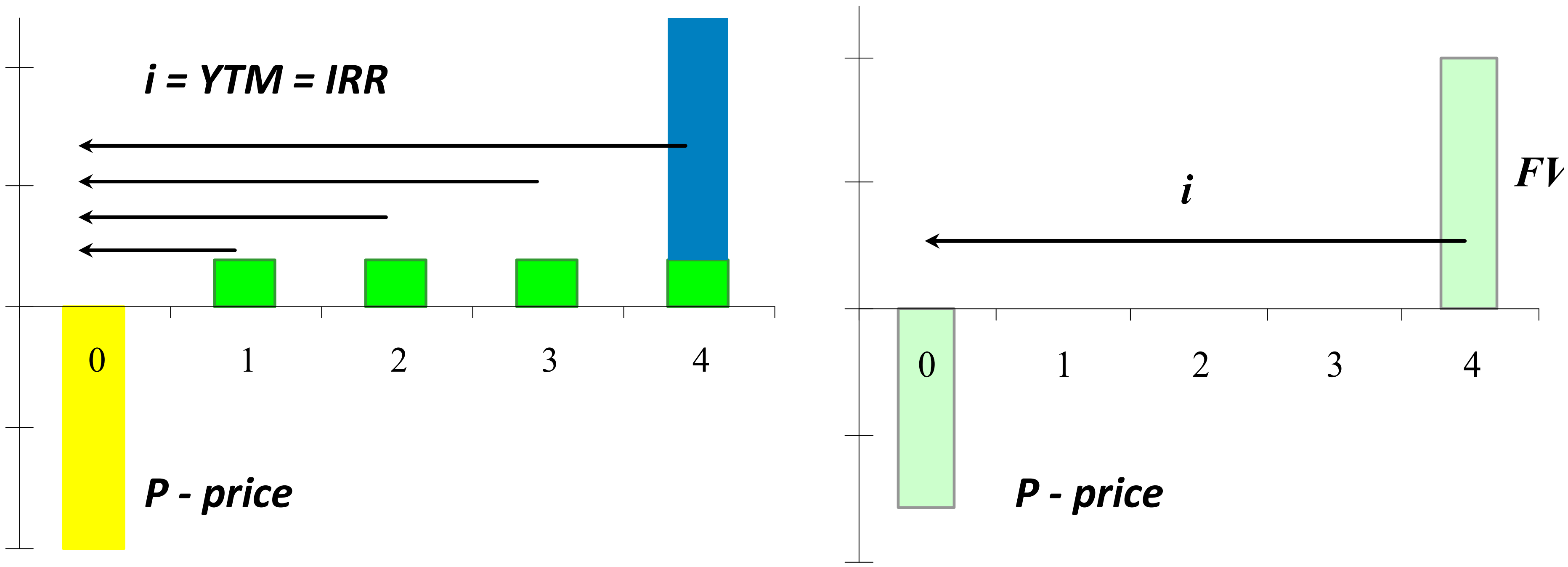
Green bond



Milestones green bond industry

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Green bond

- A green bond is a fixed-income financial instrument like any other bond. However, these bonds are issued to raise financing for climate change solutions. They can be issued by governments, banks, municipalities, or corporations in any debt format, such as private placement, securitization, and covered bonds.
- Bonds are labelled green by the issuer and should be qualified as green by an independent party

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Sustainable finance

Sustainable finance is defined as the incorporation of environmental, social, and governance (ESG) principles into business decisions, economic development, and investment strategies.

Key Pillars	Key Themes
Environment	Climate change
	Natural resources
	Pollution and waste
	Opportunities and policy
Social	Human capital
	Product responsibility
	Relations
Governance	Corporate governance
	Corporate behavior

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Application of ESG factors across Asset Classes

Asset Class	Type
Equity	Negative (exclusionary) and positive (best-in-class) approach
Debt Fixed Income	Traditional corporate bonds
	Traditional sovereign bonds
	ESG money market funds
	Green bonds
	Social bonds
	Sustainability bonds
	Green mortgage-backed securities (MBS)
Debt Bank Loans	Green loans
	Sustainability-linked loans
Alternative Investment	Green real estate investment trusts (REIT)
	Private equity (PE) and venture capital (VC)

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Application of ESG factors across Asset Classes

Debt Fixed Income	Examples
Traditional corporate bonds	Bonds with proceeds used for funding new and existing projects with environmental benefits (not labeled).
Traditional sovereign bonds	Bonds issued by governments with proceeds used for funding projects with environmental benefits (not labeled).
Green bonds	Specific bonds that are labeled green, with proceeds used for funding new and existing projects with environmental benefits.
Social bonds	Bonds that raise funds for new and existing projects that create positive social outcomes.
Sustainability bonds	Bonds with proceeds that are used to finance or refinance a combination of green and social projects.
Green mortgage-backed securities (MBS)	Green MBS securitize numerous mortgages that go toward financing green properties (Fannie Mae)

Green bonds

Green bonds are fixed income securities which finance investments with environmental or climate-related benefits (*Fu, S. Li, A.W. Ng*).

Specifically, these bonds have the following features:

- ✓ proceeds are allocated exclusively to green projects,
- ✓ proceeds are tracked and managed in a reliable manner, and
- ✓ transparency is ensured by reporting after the issuance of the bonds.

Green projects

Green bonds have been used to finance:

- ✓ climate projects,
- ✓ pollution prevention,
- ✓ sustainable agriculture,
- ✓ sustainable water management and
- ✓ other environmental initiatives.

Sustainable Development Goals

1 NO POVERTY — Waste water treatment projects

7 AFFORDABLE AND CLEAN ENERGY — Wind farms and solar PV projects, green buildings

9 INDUSTRY, INNOVATION AND INFRASTRUCTURE — Maintenance and upgrade of public transport and infrastructure projects

11 SUSTAINABLE CITIES AND COMMUNITIES — Low carbon public transport projects

12 RESPONSIBLE CONSUMPTION AND PRODUCTION — Waste management projects

13 CLIMATE ACTION — Flood protection projects

14 LIFE BELOW WATER — Renaturation projects

15 LIFE ON LAND — Sustainable agriculture and forestry projects

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Types of Green Bonds (ICMA)

Green bond issuing entities can make use of a variety of structures related to the “use of proceeds”

- ✓ Green „Use of Proceeds” Bond;
- ✓ Green „Use of Proceeds” Revenue Bond;
- ✓ Green Project Bond;
- ✓ Green Securitized Bond;

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Types of Green Bonds (ICMA)

Type	Proceeds raised by bond sale are	Debt recourse
Green "Use of Proceeds" Bonds	Earmarked for green projects	full recourse to the issue and sharing the same credit rating as the issuer.
Green "Use of Proceeds" Revenue Bonds	Earmarked for or refinances green projects	Non-recourse to the issuer and repays investors based on a revenue stream such as tolls, fees, and taxes.
Green Project Bonds	Ring-fenced for the specific underlying green project(s)	Recourse is only to the project's assets and balance sheet.
Green Securitized Bonds	portfolios of green projects or proceeds are earmarked for green projects	Recourse is to a group of Green Project(s) linked together (e.g. solar leases or green mortgages)

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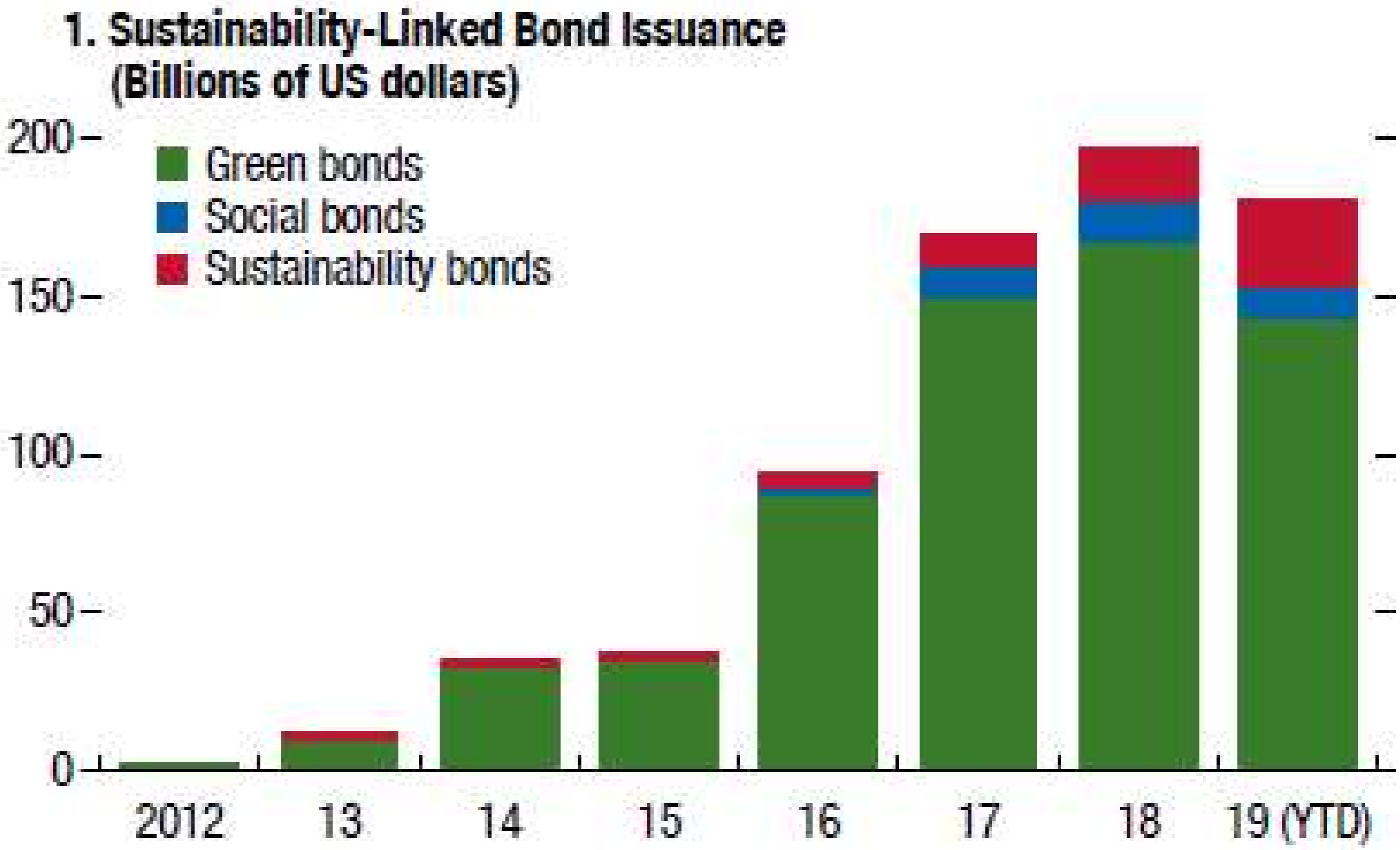
Part 2

GREEN BOND MARKET

IMF, Global Financial Stability Report.
October 2019 p.88
DOI: http://dx.doi.org/10.5089/9781498324021.082



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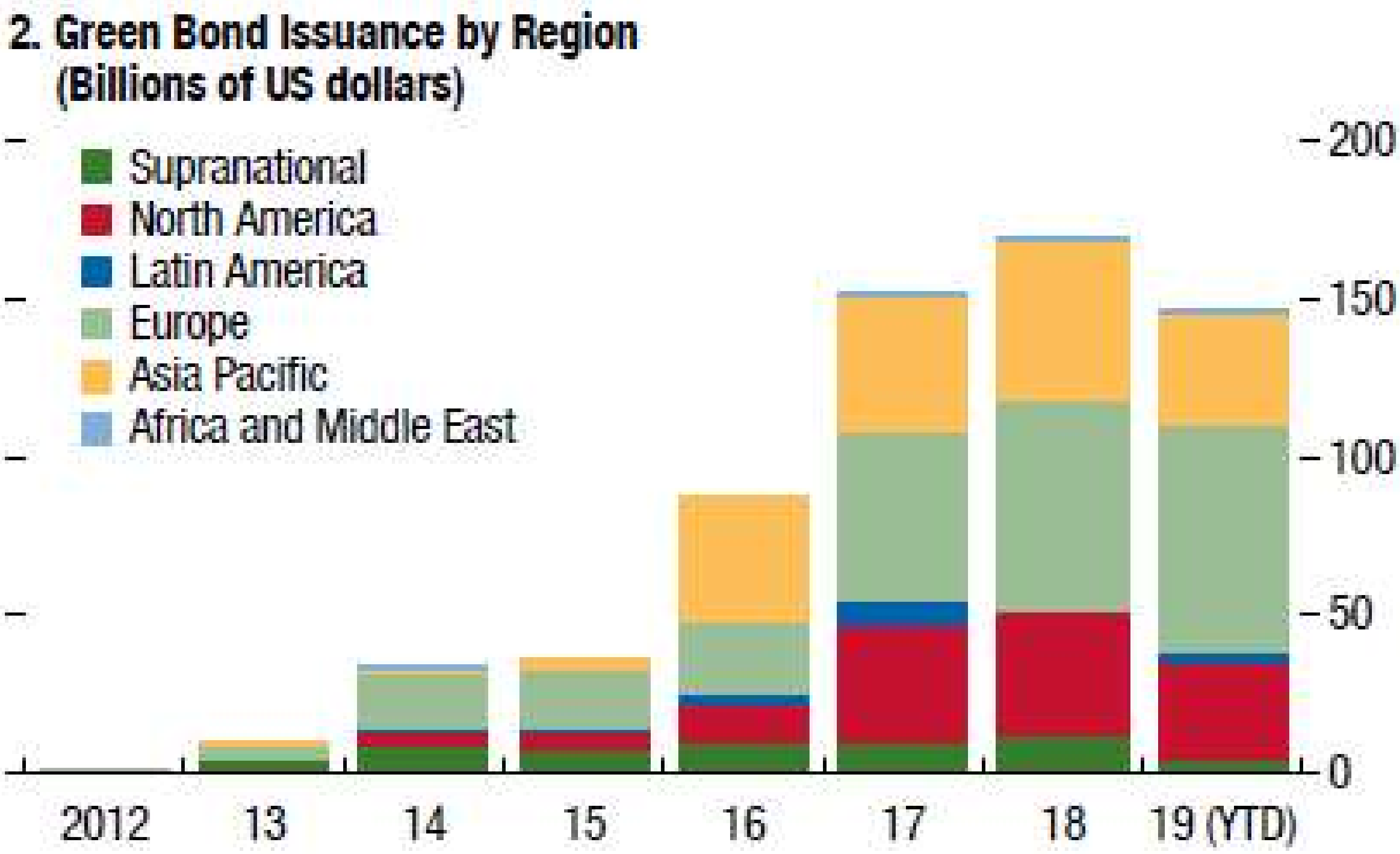

IMF, Global Financial Stability Report.
October 2019 p.88
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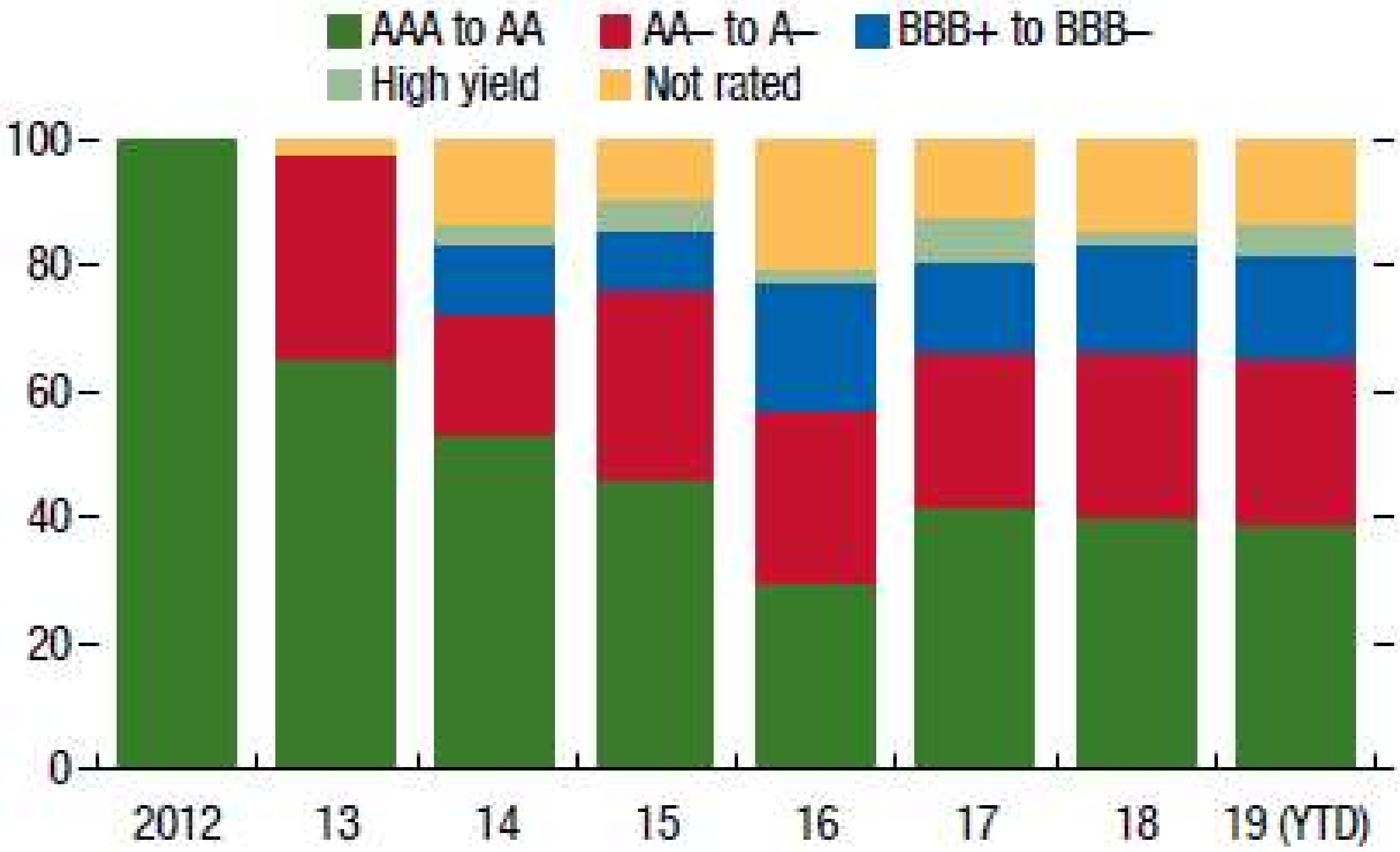
3. Green Bond Issuance by Credit Rating
(Percent of total)



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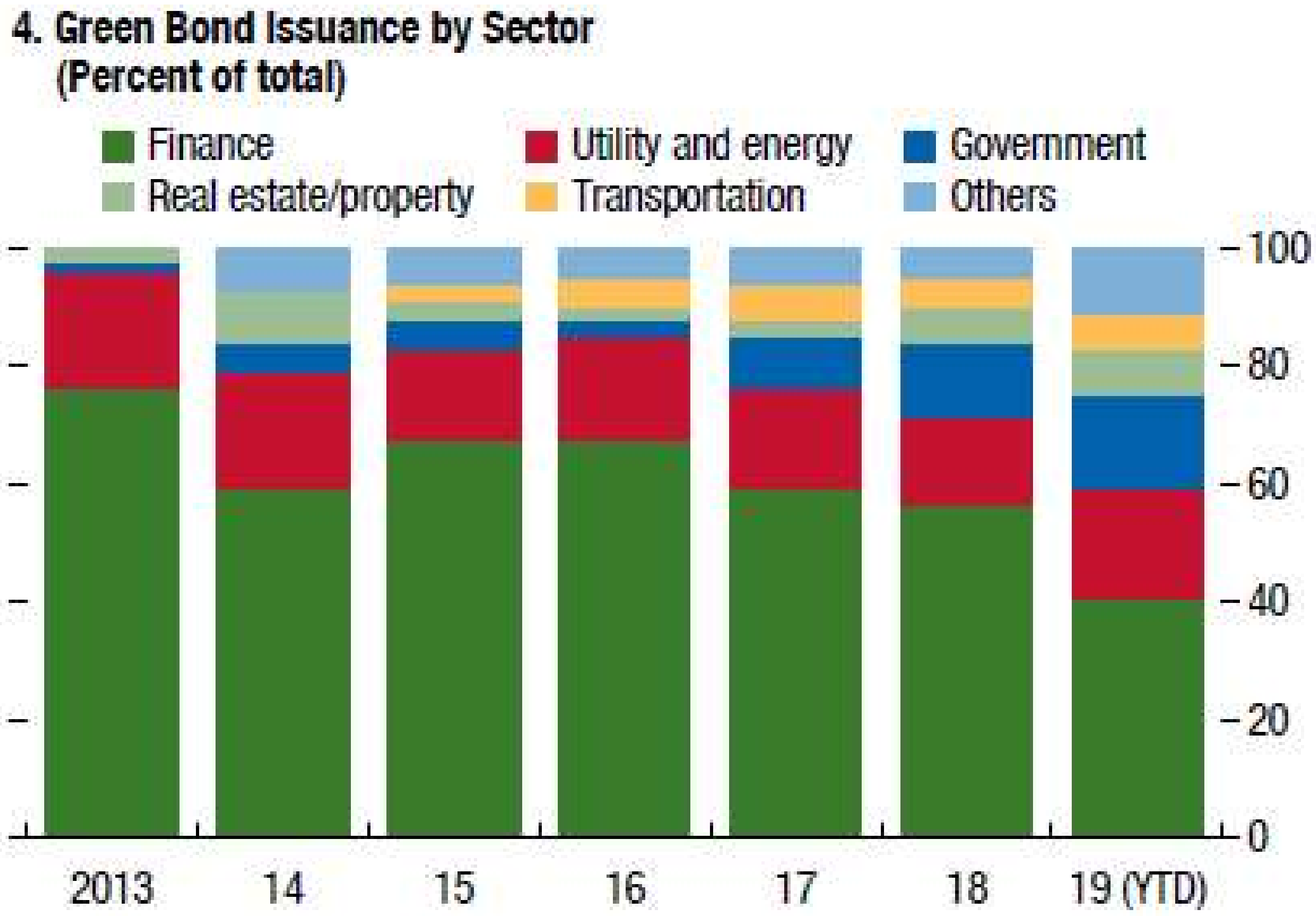

First green bond (2007)

https://www.youtube.com/watch?v=i3gIJrABLSc&index=1&list=PLuz9mCSZhLGklkMaOJTBKkiKwzoHFt6Sx

There is no standard for what kinds of activities can be funded by green bonds. Lack of common standards or criteria, causes that vast majority of green bonds are self-labeled by the issuer. (World Bank & CICERO)

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First sovereign bonds

First sovereign Green Bonds were issued in 2016 in Poland

http://pubdocs.worldbank.org/en/893761541540770521/BS-5-R-Zima-Sovereign-Green-Bonds-in-Poland.pdf

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Who decides what is „green"?

Pre-issuance reviews:

- ✓ **third party assurance** – following *Green Bond Principles* (GBP/ICMA) – checked by audit firms;
- ✓ **Second Party Opinion (SPO)** - analyzing the "greenness", providing „rating" – ESG service providers (CICERO, Sustainanalytics);
- ✓ **Green Bond Rating** – rating agencies – (Moody's, S&P)
- ✓ **Climate Bonds Certification** - *Climate Bond Standards* (Climat Bond Initiative) – CBI Certification Scheme

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Third party assurance
Green Bond Principles (GBP)

A voluntary process guidelines ICMA that recommend transparency and promote integrity in the development of the Green Bond market by clarifying the approach for issuance of a Green Bond.

The GBP have **four core components**:

(1) the use of proceeds
(2) the process for project evaluation and selection
(3) the management of proceeds
(4) Reporting

https://www.icmagroup.org/sustainable-finance/

Second Party Opinion (SPO) World Bank & CICERO

The World Bank decides what projects can be eligible for green bond proceeds based on its own selection criteria.

These criteria were reviewed by the Center for International Climate and Environmental Research University of Oslo (CICERO). CICERO also certified the International Finance Corporation's criteria for green bonds.

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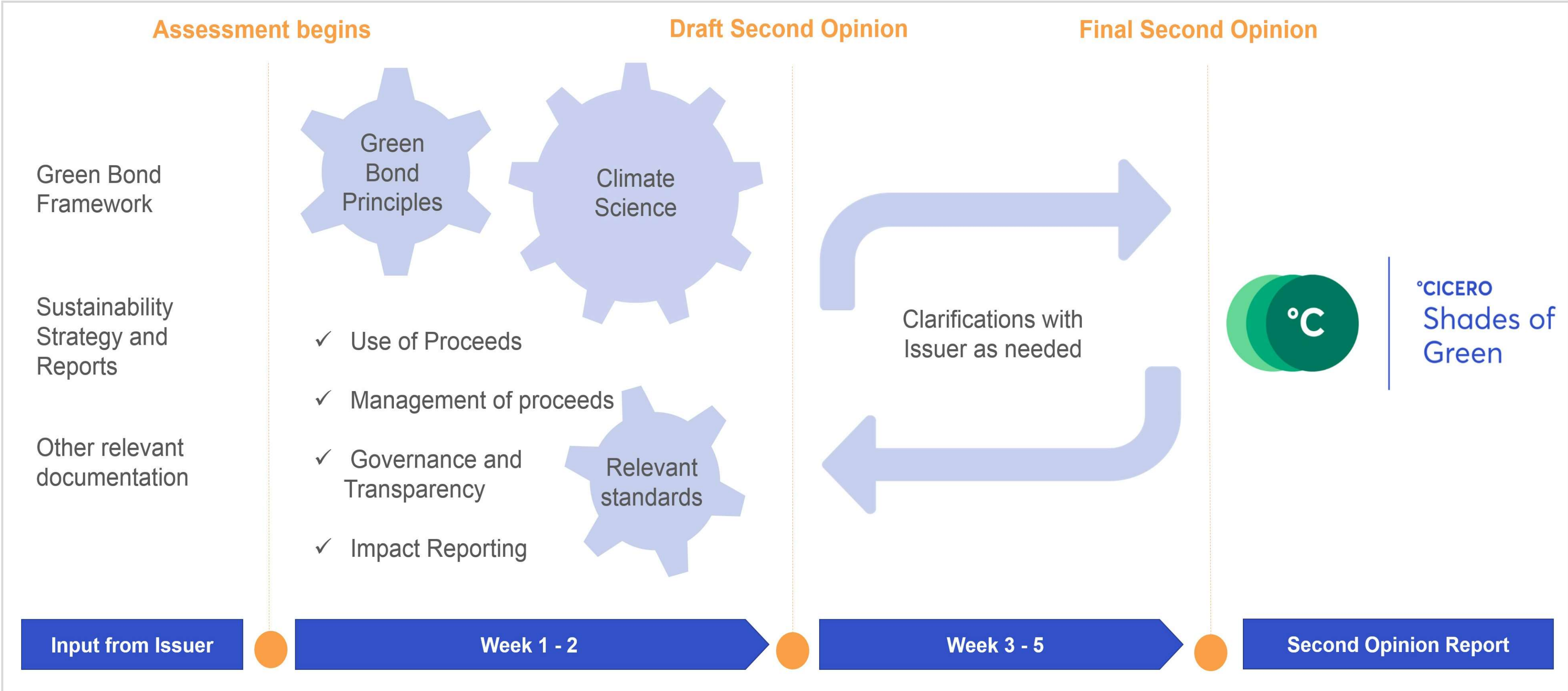
The process of getting a CICERO Second Opinion on a green bond framework



Source: https://cicero.oslo.no/en/posts/single/CICERO-second-opinions

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CICERO Shades

SHADES OF GREEN AND BROWN	EXAMPLES	
Dark green is allocated to projects and solutions that correspond to the long-term vision of a low carbon and climate resilient future.		Wind energy projects with a governance structure that integrates environmental concerns.
Medium green is allocated to projects and solutions that represent steps towards the long-term vision, but are not quite there yet.		Green buildings with a high level of certification and energy efficiency
Light green is allocated to projects and solutions that are environmentally friendly but do not by themselves represent or contribute to the long-term vision.		Hybrid personal vehicles
Light brown for efficiency improvements in projects that are associated with fossil fuel use but do not necessarily promote locking-in of emissions. Changes in the way assets are used may position them in the light green category.		Efficient fossil fuel cargo vessels
Medium brown projects can be lower emissions, but still represent risk of locking-in fossil fuel infrastructure and are exposed to risk of stranded assets.		New infrastructure for natural gas
Dark brown for the heaviest emitting projects, with the most potential for lock-in of emissions and risk of stranded assets.		New infrastructure for coal

Source: https://www.cicero.green/latestnews/2020/5/13/launching-cicero-shades-of-green-assessment-for-companies-and-equities

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Climate Bonds Certification
Climate Bonds Standards (CBS)

The Climate Bonds Initiative (CBI) developed standards for a bond to be eligible for an industry-recognized label of "Certified Climate Bond." It is a climate bonds taxonomy to define eight broad categories —Energy, transport, water, buildings, land-use and adaptation infrastructure, industry, waste management, ICT — which are then further defined with criteria, explanations and restrictions.

https://www.climatebonds.net/files/files/CBI-Taxonomy-Sep18.pdf

https://www.climatebonds.net/standard/taxonomy

https://www.climatebonds.net/files/files/cbi-gb-methodology-061020.pdf

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Last 3 months issues

[Labelled Green Bonds Data: Latest 3 Months | Climate Bonds Initiative]

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The issuers of green bonds

- ✓ **Classic Green Bond Issuers**: include only those bond issuers who allocate at least 75% of the proceeds to green assets, are aligned with the Climate Bonds Taxonomy (the standards which are used to label a bond 'Green'), and who provide sufficient information on the revenues used to finance these projects.
- ✓ **Strongly Aligned Issuers**: These include those bond issuers that derive 75%-95% of revenues from climate-aligned assets and green business lines.
- ✓ **Fully Aligned Issuers**: These include those bond issuers that derive over 95% of revenues from climate-aligned assets and green business lines.

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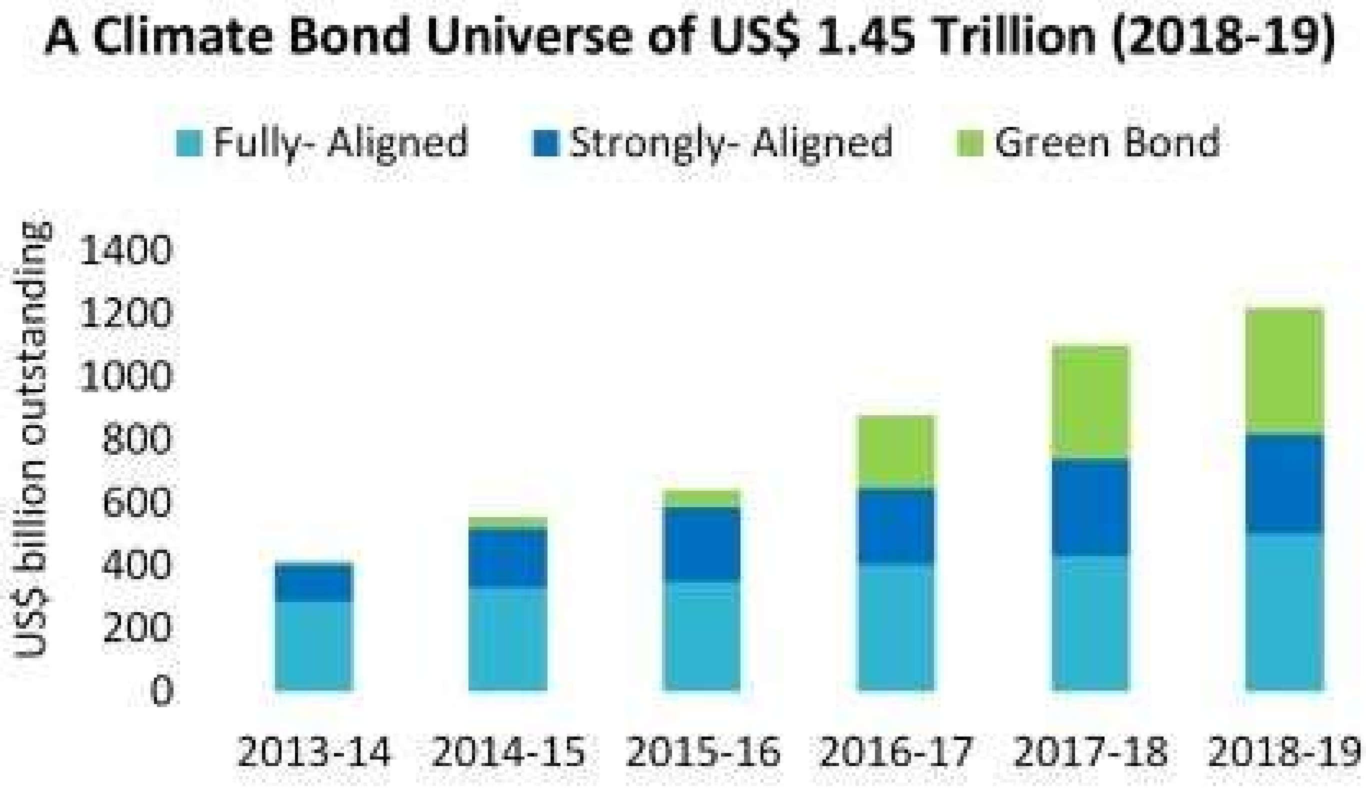
The issuers of green bonds



Source: Climate Bonds Report

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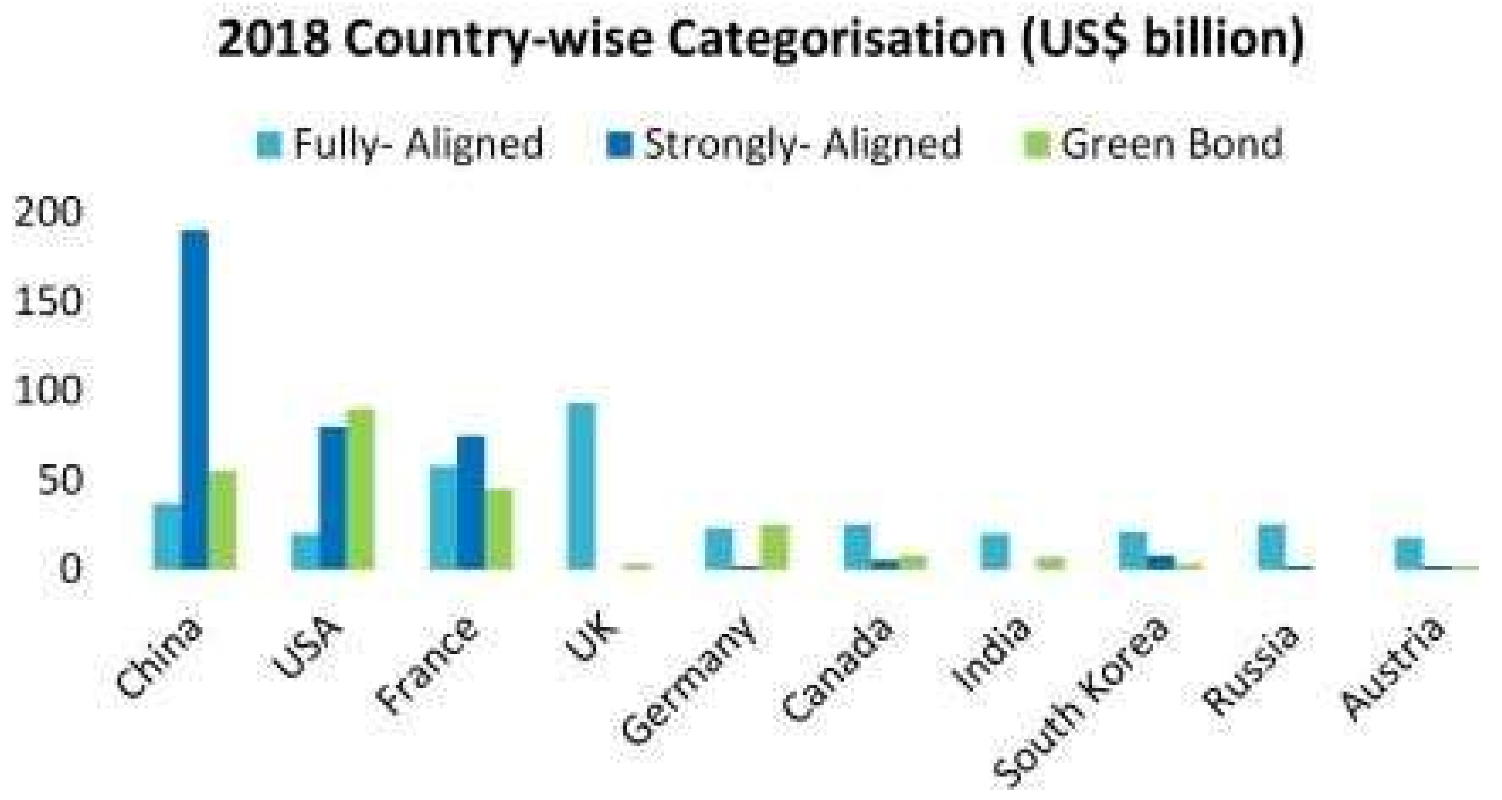
The issuers of green bonds



Source: Climate Bonds Report

EU Taxonomy

Green Projects should contribute substantially to at least one of the six environmental objectives of the EU Taxonomy Regulation:

1. Climate change mitigation;
2. Climate change adaptation;
3. Sustainable use and protection of water and marine resources;
4. Transition to a circular economy;
5. Pollution prevention and control;
6. Protection and restoration of biodiversity and ecosystems.

Examples

Example 1 – Physical assets (windfarm) Company A develops, constructs and operates windfarms. It plans to build a portfolio of six facilities, amounting 500 MW and looks for financing sources. As a windfarm is a physical green asset, Company A is looking to diversify its financing sources and plans to issue a Green Bond under the EU GBS. In addition, the financed amounts could include costs for connecting the project with grid.

Example 2 – Financial green assets (mortgage loans) A bank is looking to finance a portfolio of green real estate mortgages. The bank can do it by issuing green bonds, either not secured by these green mortgages (e.g. senior green bonds), or secured by these green mortgages (e.g. covered bonds, asset-backed securities, etc.). The green mortgages are the financial green assets, which are ultimately linked to real economy green investments.

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Green bonds benefits

<table>
<tr><th>Investors</th><th>Issuers</th></tr>
<tr><td colspan="2">Reputational benefits (e.g. marketing can highlight support for green investment);</td></tr>
<tr><td colspan="2">Articulation and enhanced credibility of sustainability strategy (“money where your mouth is”);</td></tr>
<tr><td>Lack of additional risk, green bonds can be incorporated into pension funds’ existing asset allocations;</td><td>Access to “economies of scale” as majority of issuance costs are in setting up the processes;</td></tr>
<tr><td>Improving diversification of bond issuer base;</td><td>Improving diversification of bond investor base;</td></tr>
</table>

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Related financial risks

Physical risks - arise from damage to property, land, and infrastructure from catastrophic weather-related events and broader climate trends;

Transition risks - arise from changes in the price of stranded assets and broader economic disruption because of evolving climate policy, technology, and market sentiment during the adjustment to a lower-carbon economy.

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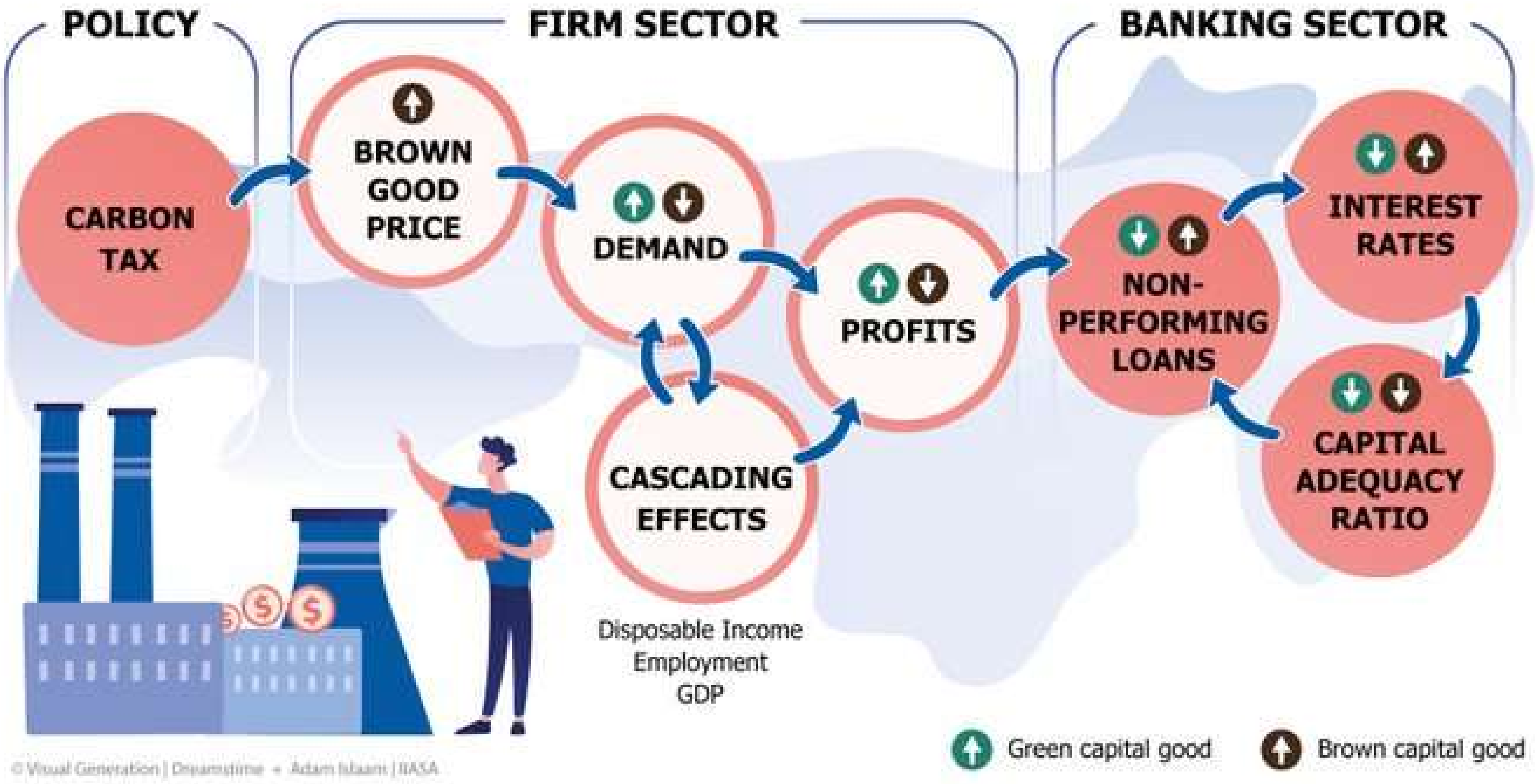
What we expect



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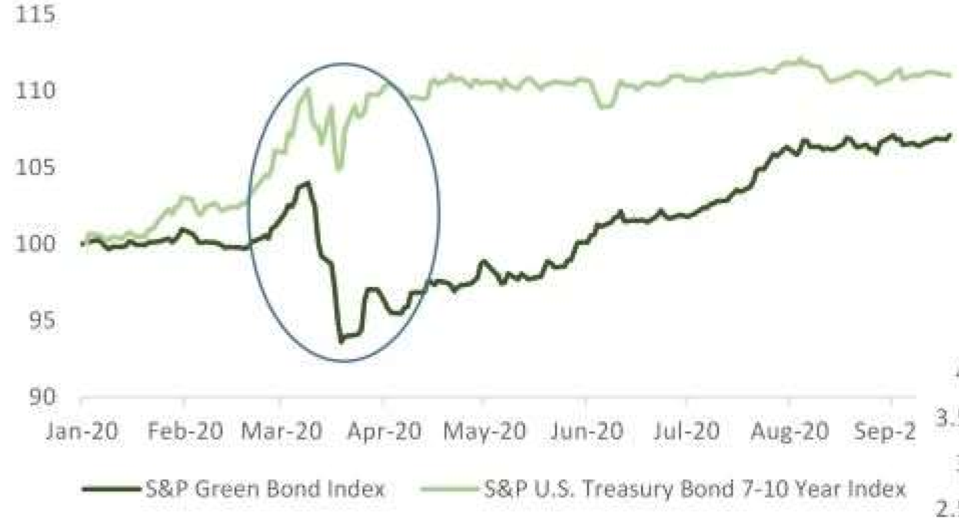
Historical performance

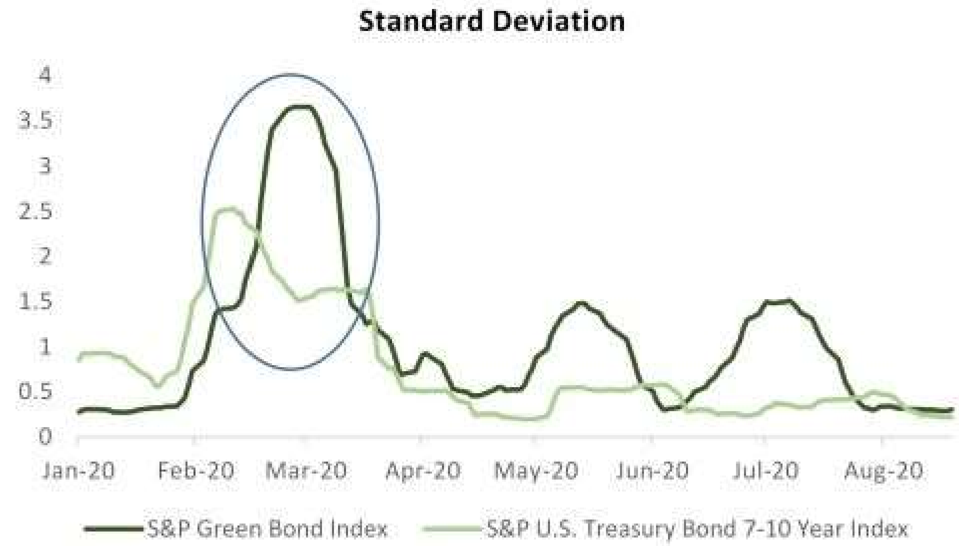
	Green Bonds	Treasury Bonds 7-10 Y
1-year Return	7.92%	11.04%
5-year Return	3.78%	4.63%
10-year Return	2.52%	4.46%

Source: S&P Global

Covid 19 consequences



Source: S&P Global



Source: S&P Global

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Why the green yield is lower?

- ✓ Fed Stimulus Program, UK Government's International Climate Fund (ICF), steps taken by the Chinese government to achieve the long-term goal being carbon-neutral
- ✓ exponential increase in Green Bond issues
- ✓ strong demand for these bonds among green investors who are increasingly becoming enthusiastic about the ESG style of investing in this era of climate change.
- ✓ green bonds have a positive correlation of 0.304 with T-Bonds and 0.647 with the S&P 500.

1Q – Green swan?

Carbon-intensive companies, such as oil and gas companies and car manufacturers, are typically also capital intensive and thus issue more corporate bonds. By taking assets proportional to the market, the ECB's asset portfolio is skewed towards high-carbon companies relative to low-carbon companies. The carbon intensity (defined as carbon emissions divided by sales) of the ECB's corporate bond portfolio is 57 percent higher than the average carbon intensity of EU companies. This large carbon bias makes the ECB a brown central bank.

https://www.bruegel.org/2021/02/a-brown-or-a-green-european-central-bank/

https://voxeu.org/article/brown-assets-might-be-next-subprime

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2Q – Green Finance: Solution or Trap?

In a capitalist society, we tend to recognize things only when there is an economic value attached to them. This has led to the creation of green bonds.

Types of green finance	Elements of (green) finance	Forms of regulation	Implications
Neoliberal green finance	Private green finance	Voluntary standards (CSR, ESG)	No or very limited positive environmental effects legitimising finance, opposing general binding environmental rules
	Private green finance supported by public money	Subsidies, including public risk taking (guarantees)	Transfer of public finance to private (finance)

https://www.mattersburgerkreis.at/dl/musLJMJKONOJqx4KooJK/JEP_2020_4_2_J_ger_Schmidt_Global_Political_of_Green_Finance_Regulationist.pdf

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Part 3

BOND'S PRICE and YIELD RELATION

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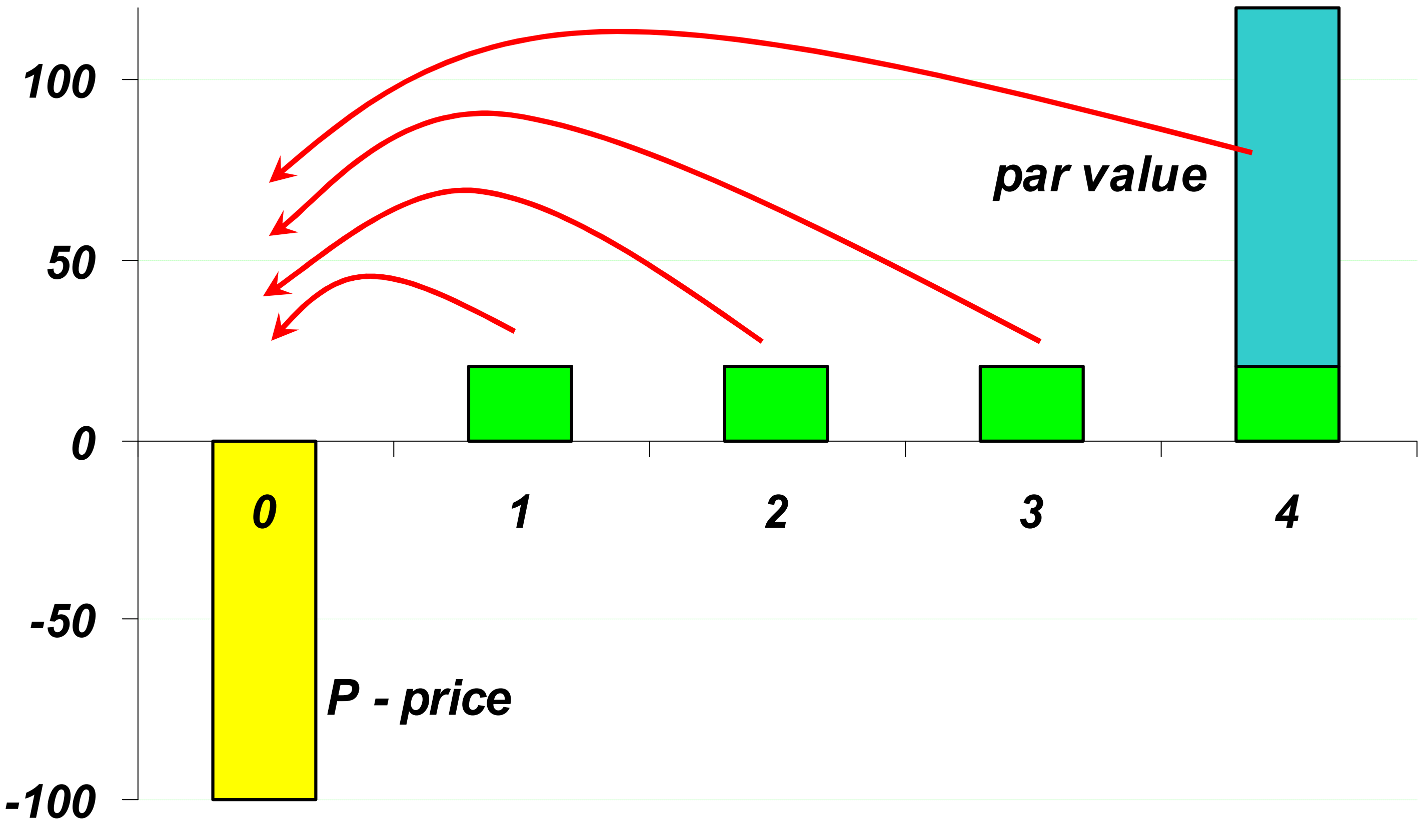
Purchase price of bond



Purchase price of bond

$$P = FV + FV \cdot (i_{cp} - i)\frac{1-(1+i)^{-N}}{i}$$

where : P – purchase price of bond

FV – face value, par value

i_{cp} – coupon rate

i – yield rate

N – number of payments (=*n for annually payments*)

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$$P = FV + FV \cdot (i_{cp} - i)\frac{1-(1+i)^{-N}}{i}$$

(1). If the investor wants bigger rate then the issuer offers:

$$i > i_{cp} \quad \Rightarrow \quad (i_{cp} - i) < 0 \quad and \quad P < FV$$

(2). If: $i < i_{cp} \quad \Rightarrow \quad (i_{cp} - i) > 0 \quad and \quad P > FV$

(3). If: $i = i_{cp} \quad \Rightarrow \quad (i_k - i) = 0 \quad and \quad P = FV$

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Zero-coupon bond

$$P_{zero} = \frac{FV}{(1+i)^n}$$

where : P- price of the bond

FV - nominal

i –yield to maturity

n – time to maturity **in years**

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Example

Issuer	PepsiCo
Issuance date	October 2019
Nominal value	$1 billion
Nominal currency	USD
Rating (issuer, bond)	A+ (S&P), A (Moody's)
Framework	Green bond
Tenure	30 years
Coupon	2.875%
Use of proceeds	Eco-friendly plastics, water use efficiency, packaging, and cleaner transportation
Bookrunners	Morgan Stanley, Goldman Sachs, Mizuho Financial group

https://sec.report/Document/0001047469-19-005653/0001047469-19-005653.txt#ds41801_description_of_debt_securities

Example

Issuer	U.S. State of Massachusetts
Issuance date	September 2014
Nominal value	$350 million
Nominal currency	USD
Rating (issuer, bond)	AA+ (Fitch), Aa1 (Moody's), AA+ (S&P)
Framework	Green bond
Tenure	3 to 17 years
Coupon	2.45%
Subscription level	3 times
Investor base	Residents and local retail investors
Use of proceeds	Water projects, offshore wind port facilities, energy-efficient buildings, and restoration and preservation projects
Bookrunners	Morgan Stanley

https://www.climatebonds.net/files/files/DC%20Water%20case%20study%20-%20final%281%29.pdf

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COUPON CALCULATION of a NEW ISSUE

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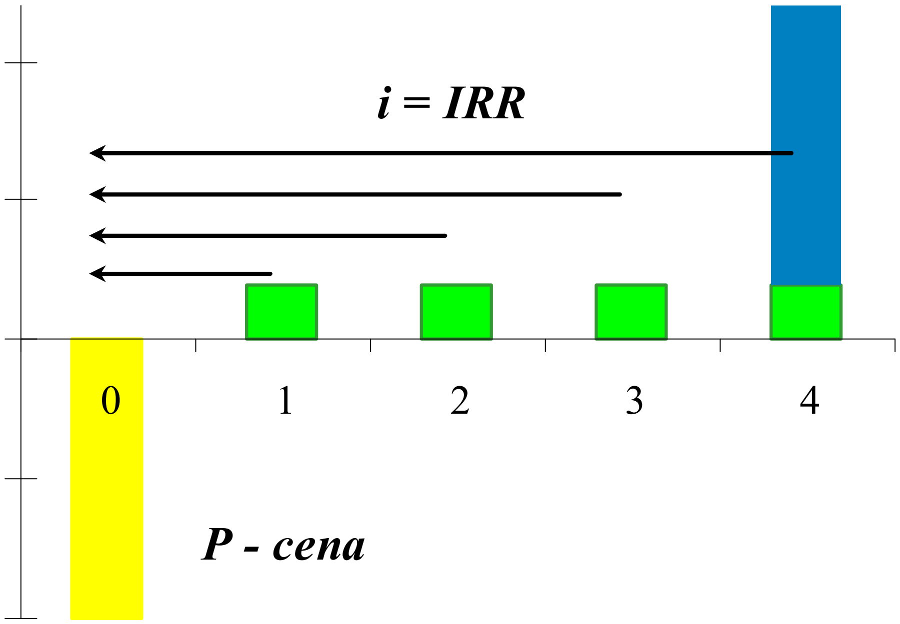
YTM for a coupon bond



$$YTM = IRR$$

where :

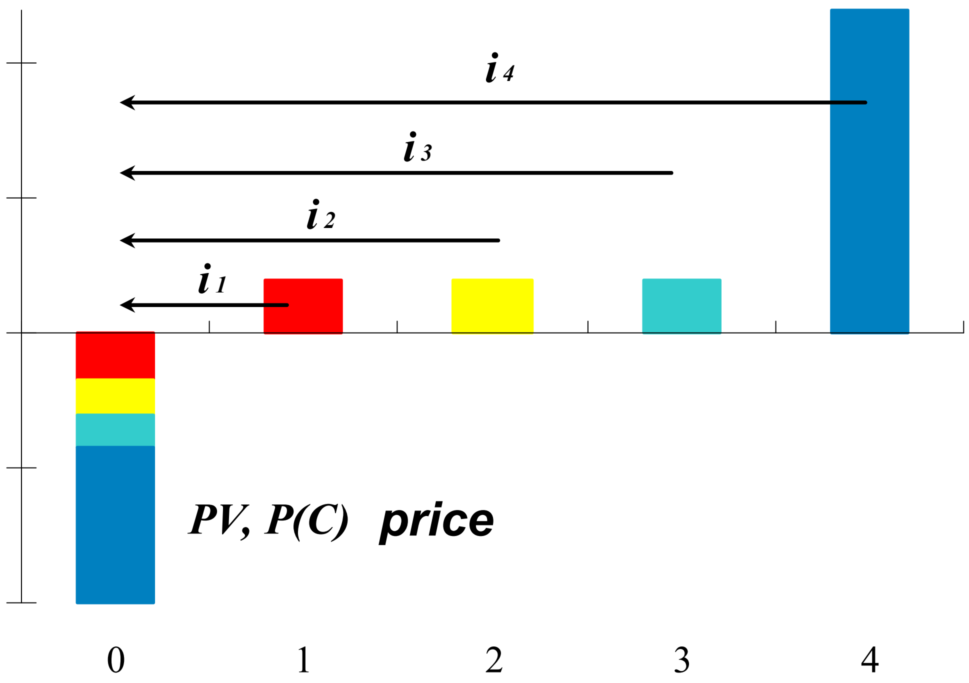
P – bond's price

N - nominal

i – desired interest rate

n – time to maturity **in years**

Coupon bond as a sum (portfolio) of zero-coupon bonds



where :

P – bond’s price

N - nominal

i – desired interest rate

n – time to maturity **in years**

$$d(t_j) = \begin{cases} \dfrac{1}{(1+i_j)^{t_j}} \end{cases}$$

$$\delta(\tau_j) = e^{-i(\tau_j)\cdot\tau_j}$$

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Coupon bond as a sum (portfolio) of zero-coupon bonds

For a compound interest:

$$P(C)=\frac{C}{(1+i_1)^{t_1}}+\frac{C}{(1+i_2)^{t_2}}+\cdots+\frac{C+N}{(1+i_n)^{t_n}}$$

$$P(C)=C\cdot d(t_1)+C\cdot d(t_2)+\cdots+(C+N)\cdot d(t_n)$$

Coupon of a new issue:

$$C=\frac{P(C)-Nd(t_n)}{d(t_1)+d(t_2)+\cdots+d(t_n)}$$

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Coupon bond as a sum (portfolio) of zero-coupon bonds

For continuously compound interest:

$$P(C) = Ce^{-i(\tau_1)\cdot\tau_1} + Ce^{-i(\tau_2)\cdot\tau_2} + \ldots + (C+N)e^{-i(\tau_n)\cdot\tau_n}$$

$$P(C) = C\cdot\delta(\tau_1) + C\cdot\delta(\tau_2) + \cdots + (C+N)\cdot\delta(\tau_n)$$

Coupon of a new issue:

$$C = \frac{P(C) - N\delta(\tau_n)}{\delta(\tau_1) + \delta(\tau_2) + \cdots + \delta(\tau_n)}$$

Example

The government wants to set an annual coupon rate for a 5-year bond (nominal is 100). Find the coupon if the following spot rates are available on the market: 1-year equal to 3.5%, 2-year 3.7%, 5-year 3.55%, 10-year 3.45%.

Find the coupon for a 5 year issue.

The equation of the discount function is:

$$d(t) = 0{,}0007 \cdot t^2 - 0{,}0355t + 1$$

$$C = \frac{P(C) - N \cdot d(t_n)}{d(t_1) + d(t_2) + \cdots + d(t_n)} = \frac{100 - 100 \cdot 0{,}84}{4{,}51} = 3{,}55$$

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Part 5

GREEN BOND YIELD CURVE

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Rate interrelation

$$P(\tau) = \delta(\tau) = e^{-\,i(\tau)\cdot\tau} = e^{-\int_0^{\tau} f(m)\,dm}$$

where : *δ(τ)*- discount factor

i(τ)- zero-coupon rate

f(τ)- forward rate

τ – term to maturity

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Coupon bond as a sum (portfolio) of zero-coupon bonds

$$P(C) = C \cdot \delta(\tau_1) + C \cdot \delta(\tau_2) + \cdots + (C + N) \cdot \delta(\tau_n)$$

$$P(C) = P(\tau_1) + P(\tau_2) + \cdots + P(\tau_n)$$

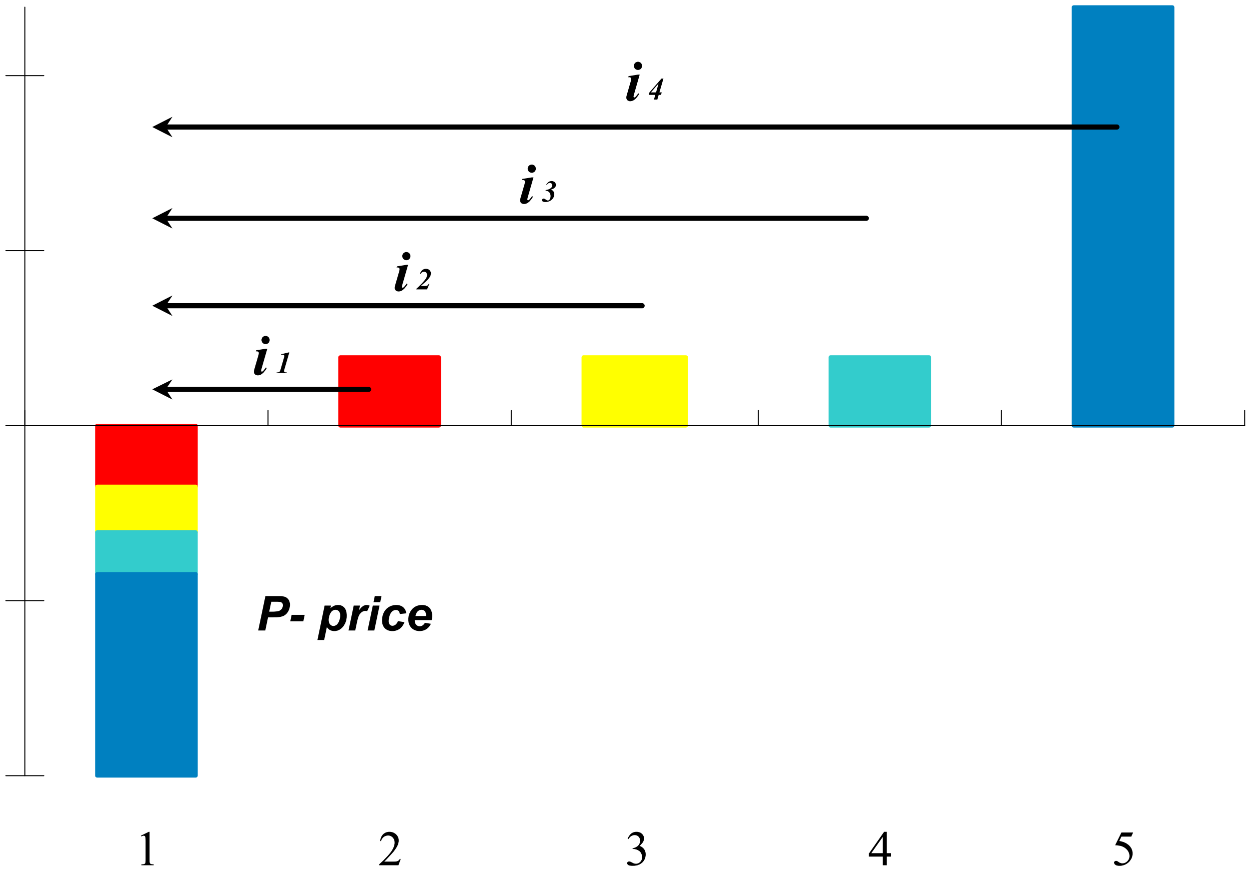
where : *δ(τ)*- discount factor

i(τ)- zero-coupon rate

f(τ)- forward rate

τ – term to maturity

Spot rates



Yield construction

1. Cash flow matrix building
2. Exemplification that theoretical prices’ array is tantamount to cash flow matrix multiplied by discount factors’ column

$$\begin{bmatrix} \overline{P_1} \\ \overline{P_2} \\ \dots \\ \overline{P_k} \end{bmatrix} = C \cdot \begin{bmatrix} \overline{\delta}(\tau_1) & \overline{\delta}(\tau_2) & \dots & \overline{\delta}(\tau_m) \end{bmatrix}^T$$

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Yield construction

3. Find such parameters $\beta_{0}, \beta_{1}, \beta_{2}, \beta_{3}, \tau_{1}, \tau_{2}$ *that*:

$$\frac{\sum_{i=1}^{k}(P_i - \overline{P_i})^2}{k} \to \min$$

$$\begin{bmatrix} \overline{P_1} \\ \overline{P_2} \\ \ldots \\ \overline{P_k} \end{bmatrix} = C \cdot \begin{bmatrix} \overline{\delta}(\tau_1) & \overline{\delta}(\tau_2) & \ldots & \overline{\delta}(\tau_m) \end{bmatrix}^T$$

$$\overline{\delta}(\tau_m) = e^{i(\tau_m)\tau_m}$$

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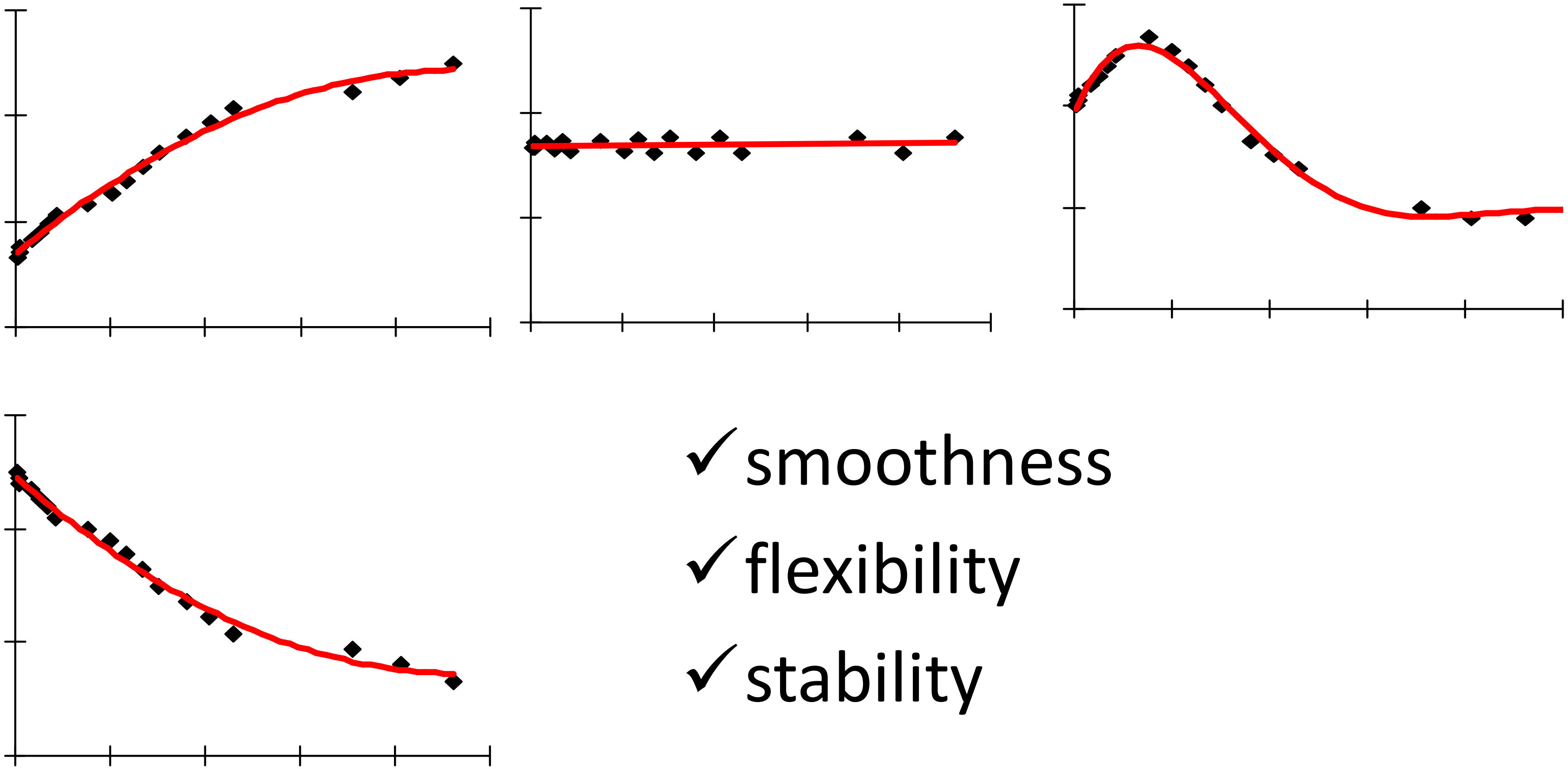
Shapes of the yield curve



✓smoothness

✓flexibility

✓stability

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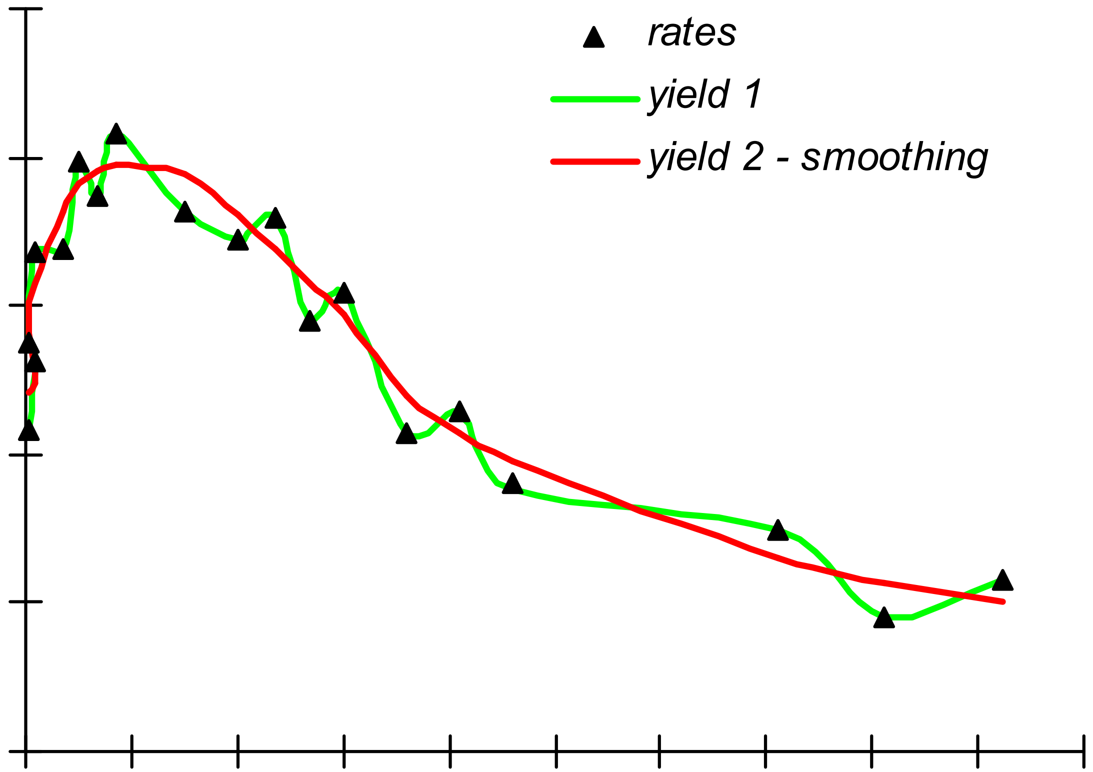
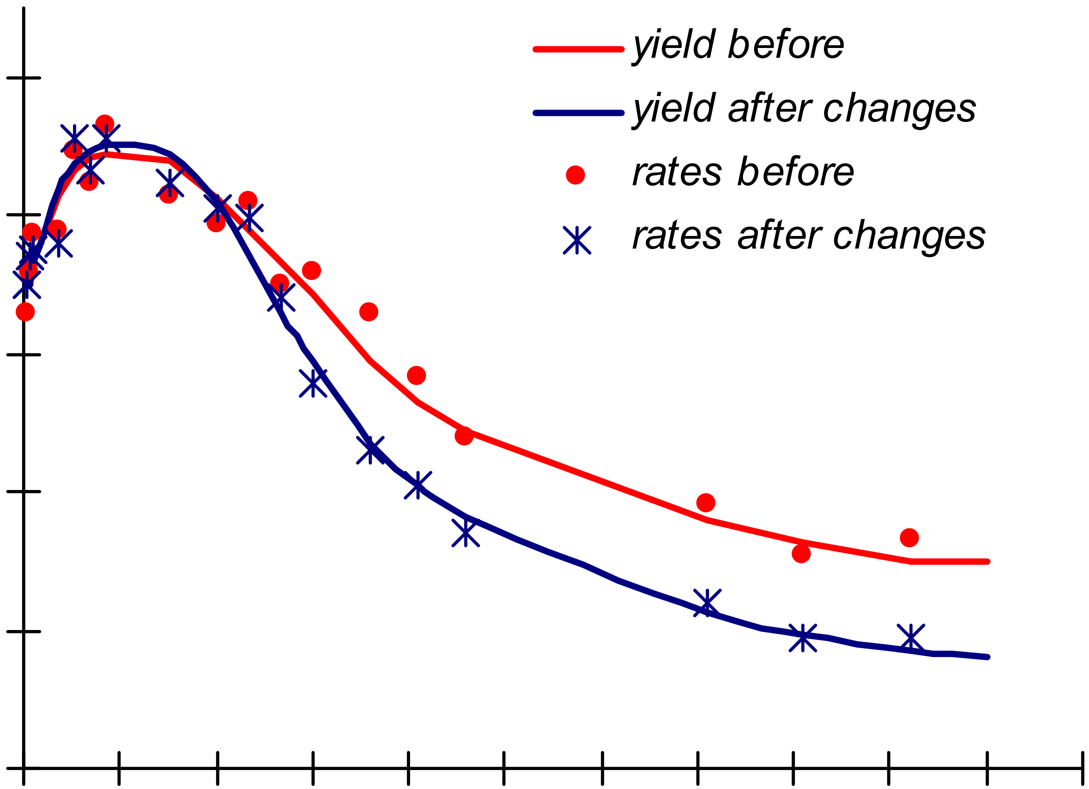
Yield curve - smoothness

- No oscillations



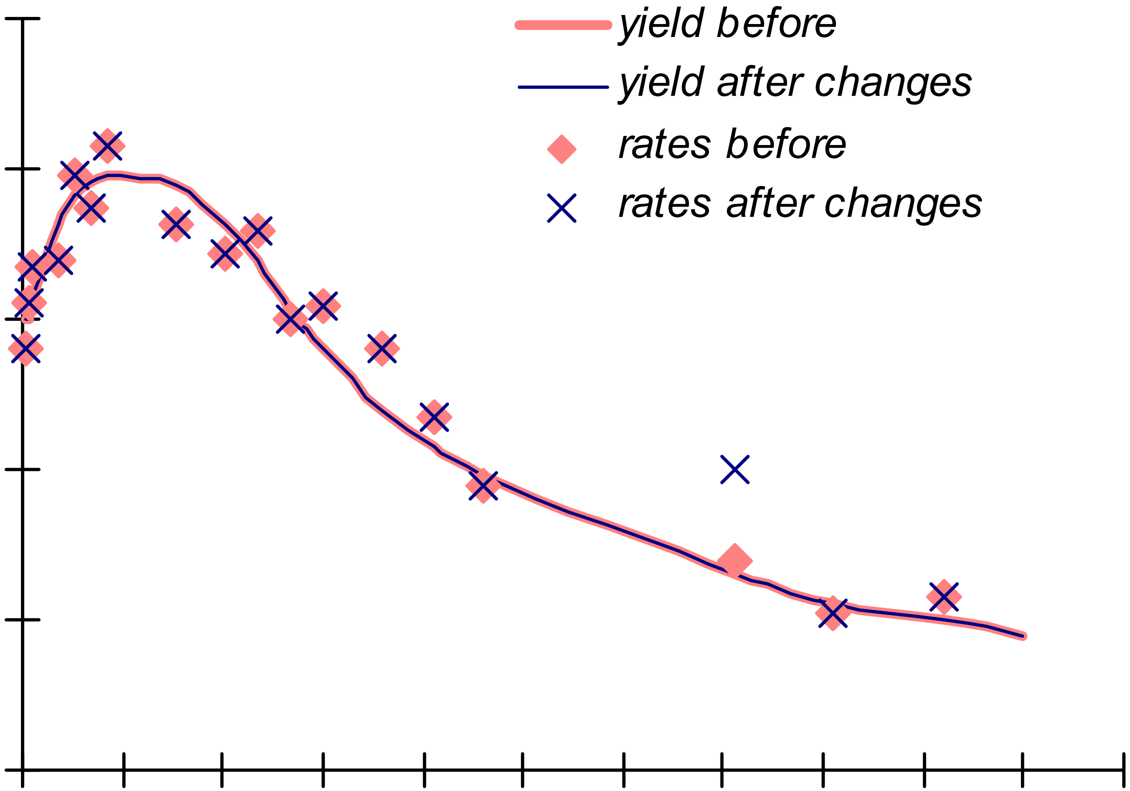
Yield curve - flexibility

- The yield could fit the date easily



Yield curve – stability

- The yield is not sensitive to small changes in data



Construction models

Parametric models:

a) Nelson-Siegel

b) Svensson

Cubic-splines models

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Nelson-Siegel model

$$f(\tau) = \beta_0 + (\beta_1 + \beta_2 \frac{\tau}{v_1}) \cdot e^{-\frac{\tau}{v_1}}$$

$$i(\tau) = \beta_0 + (\beta_1 + \beta_2) \frac{1 - e^{-\frac{\tau}{v_1}}}{\frac{\tau}{v_1}} - \beta_2 \cdot e^{-\frac{\tau}{v_1}}$$

where : *f(τ)*- instantaneous forward rate $f(\tau) = f(t; t+\tau)$

τ – term to maturity in years

$\beta_0, \beta_1, \beta_2, v_1$ - estimated parameters

i(τ)- zero coupon yield

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Svensson model

$$f(\tau) = \beta_0 + (\beta_1 + \beta_2 \frac{\tau}{v_1}) \cdot e^{-\frac{\tau}{v_1}} + \beta_3 \frac{\tau}{v_2} \cdot e^{-\frac{\tau}{v_2}}$$

$$i(\tau) = \beta_0 + (\beta_1 + \beta_2) \frac{1 - e^{-\frac{\tau}{v_1}}}{\frac{\tau}{v_1}} - \beta_2 \cdot e^{-\frac{\tau}{v_1}} + \beta_3 \left(\frac{1 - e^{-\frac{\tau}{v_2}}}{\frac{\tau}{v_2}} - e^{-\frac{\tau}{v_2}} \right)$$

where : *f(τ)*- instantaneous forward rate

τ – term to maturity in years

$\beta_0, \beta_1, \beta_2, \beta_3, v_1, v_2$ - estimated parameters

i(τ)- zero coupon yield

Parameters interpretation

$$\lim_{\tau \to \infty} f(\tau) = \beta_0 \qquad \lim_{\tau \to 0} f(\tau) = \beta_0 + \beta_1$$

- $\beta_0 > 0$
- if $\beta_1 > 0$ that the curve is negative
- if $\beta_1 < 0$ that the curve is positive
- $v_1 \, v_2 > 0$
- if $\beta_2 > 0$ i $\beta_3 < 0$ that the curve has shape "∩∪"
- if $\beta_2 < 0$ i $\beta_3 > 0$ that the curve has shape "∪∩"

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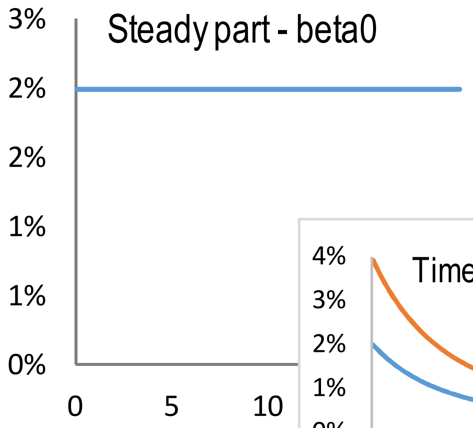
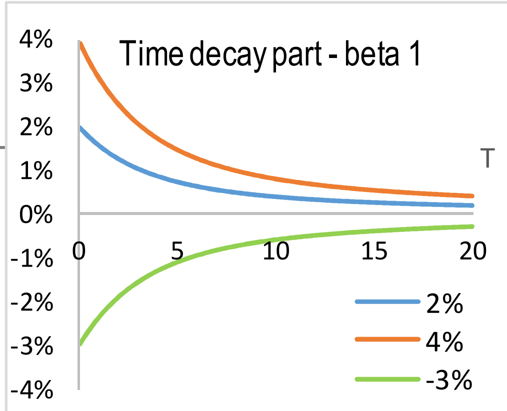
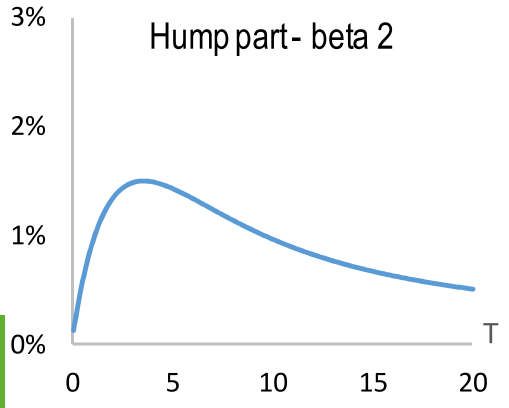
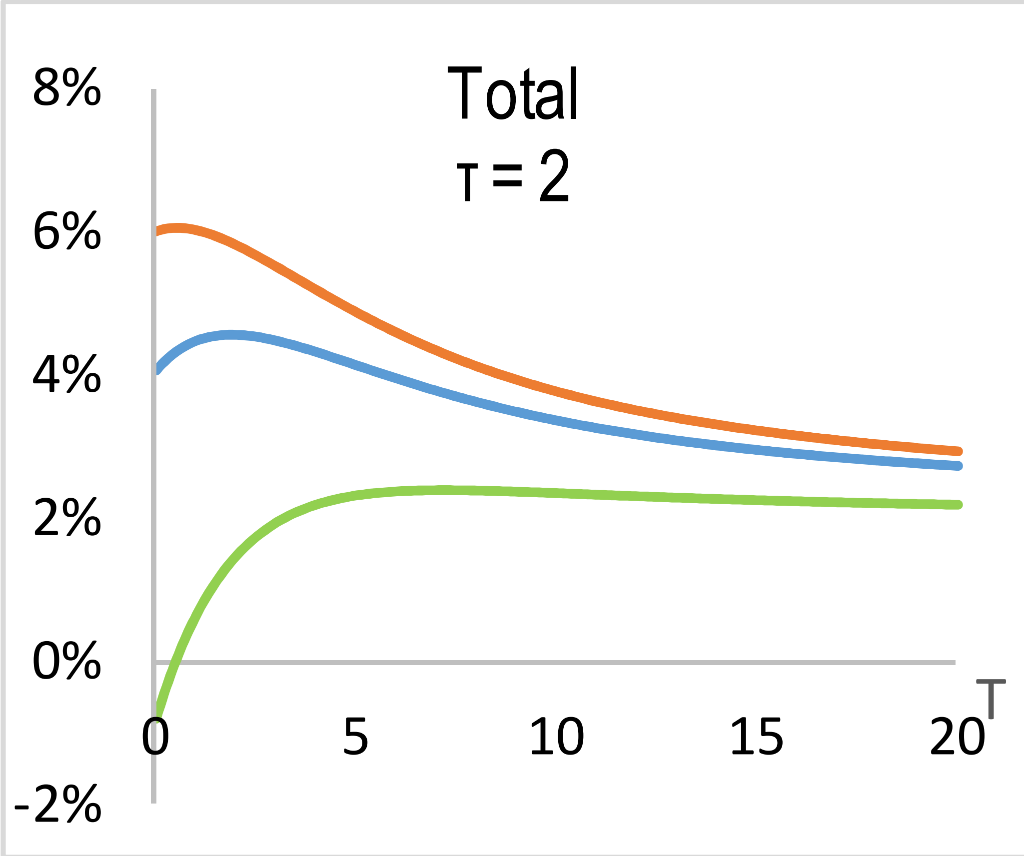
Forward yield (N-S)

(dependence on β_1)









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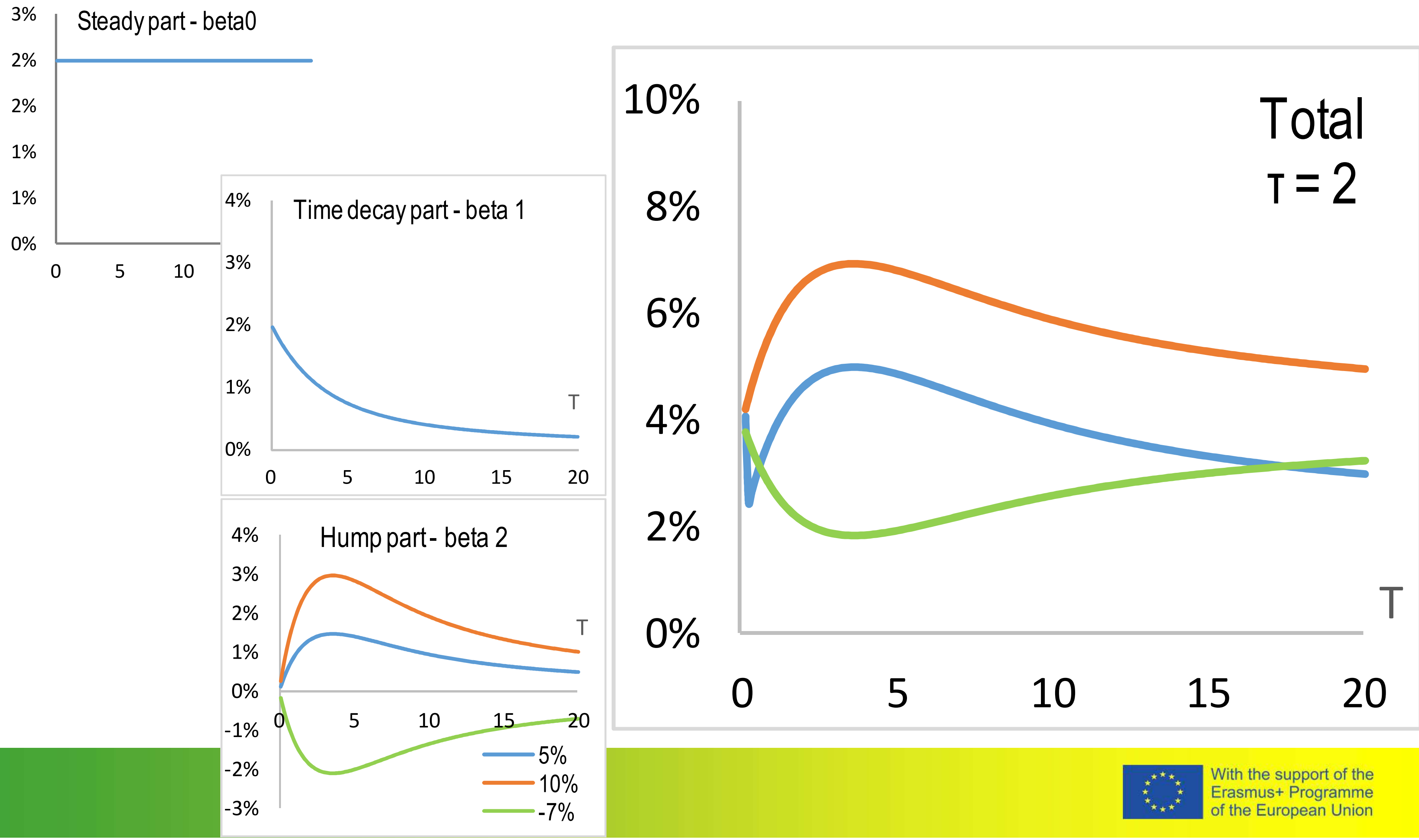
Forward yield (N-S)
(dependence on β_2)



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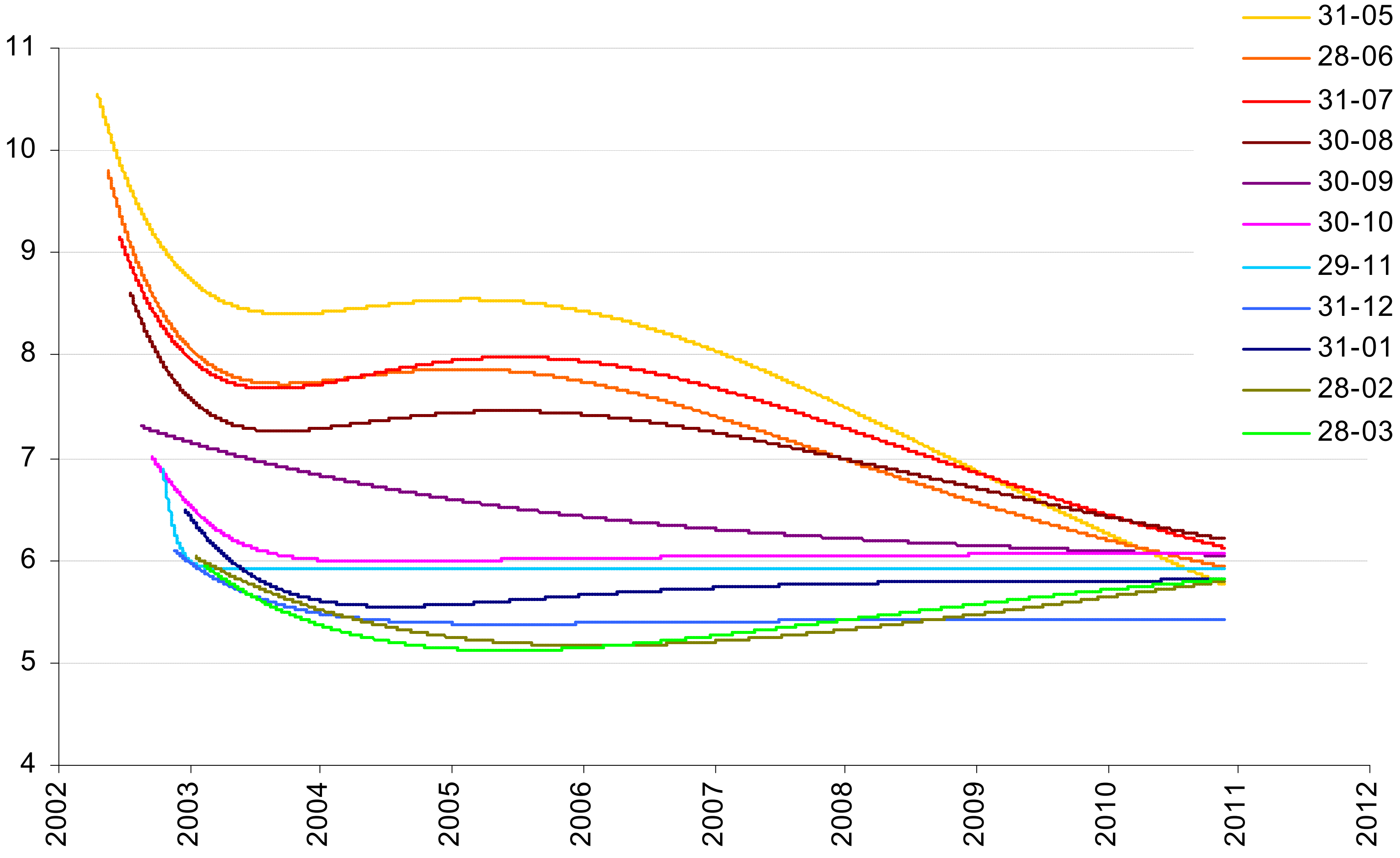
Spot rate estimation (Sv)



https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html

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Parsimonious models pros/cons

- forward rate smoothness
- flexibility – to capture movements
- stability – small changes in the data

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Splines models

$$\delta(\tau) = \sum_{j=1}^{k} \gamma_j B_j(\tau)$$

where : *δ(τ)- discount factor*

B_j – base polynomial B-spline

γ_j – parameters to be estimated

k – numbers of base parameters equal to n+3
(n – number of polynomial creating spline)

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B-spline

$$B_j^r(x) = \frac{B_j^{r-1}(x)(x - x_j) + B_{j+1}^{r-1}(x)(x_{j+1+r} - x)}{(x_{j+1+r} - x_j)}$$

where: $B_j^r(x)$ - *r*-order *B-spline* for *j=1,2,...n+3*

but:

$$B_j^0(x) = \begin{cases} 1 & for \quad x_j < x < x_{j+1} \\ 0 & otherwise \end{cases}$$

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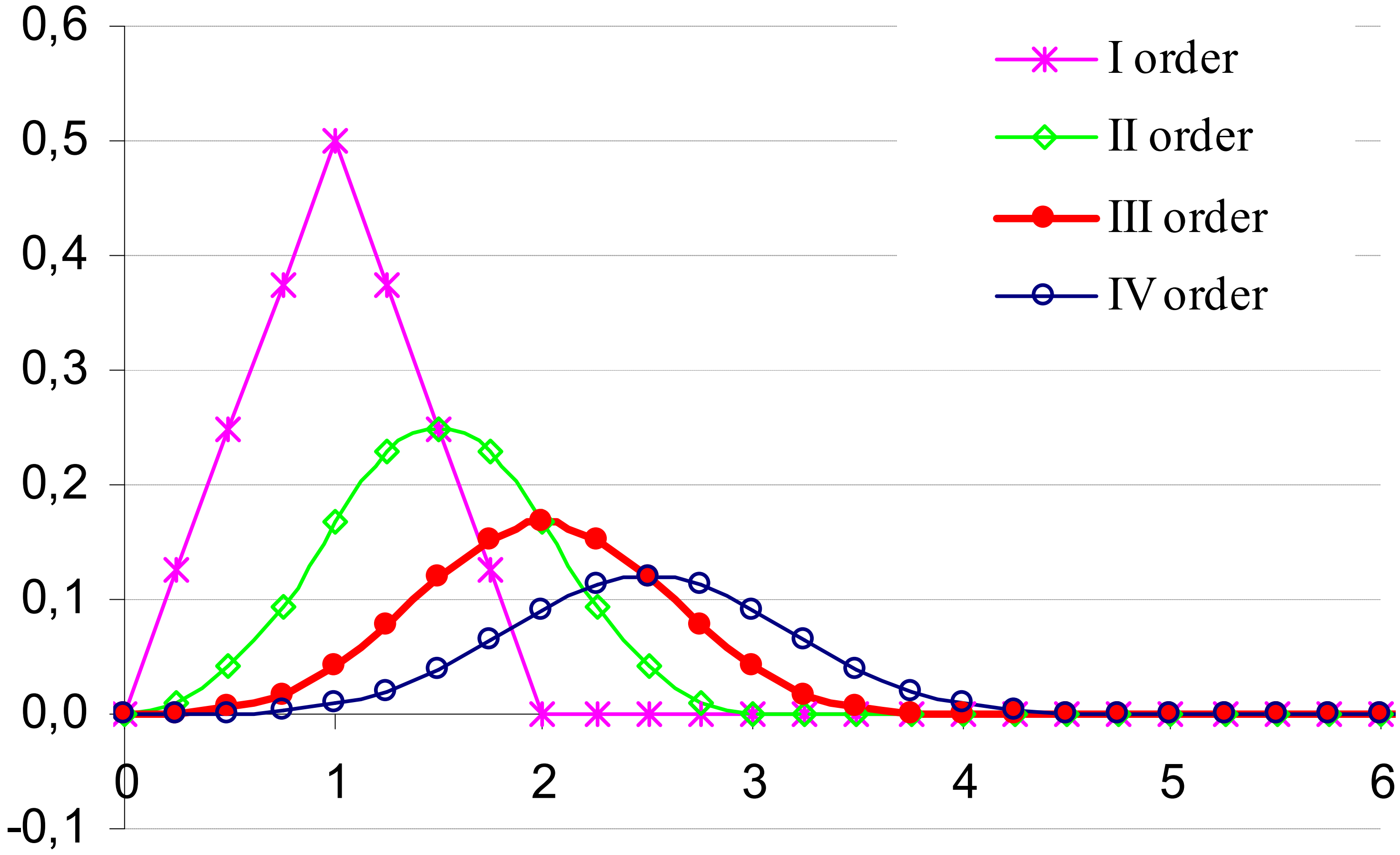
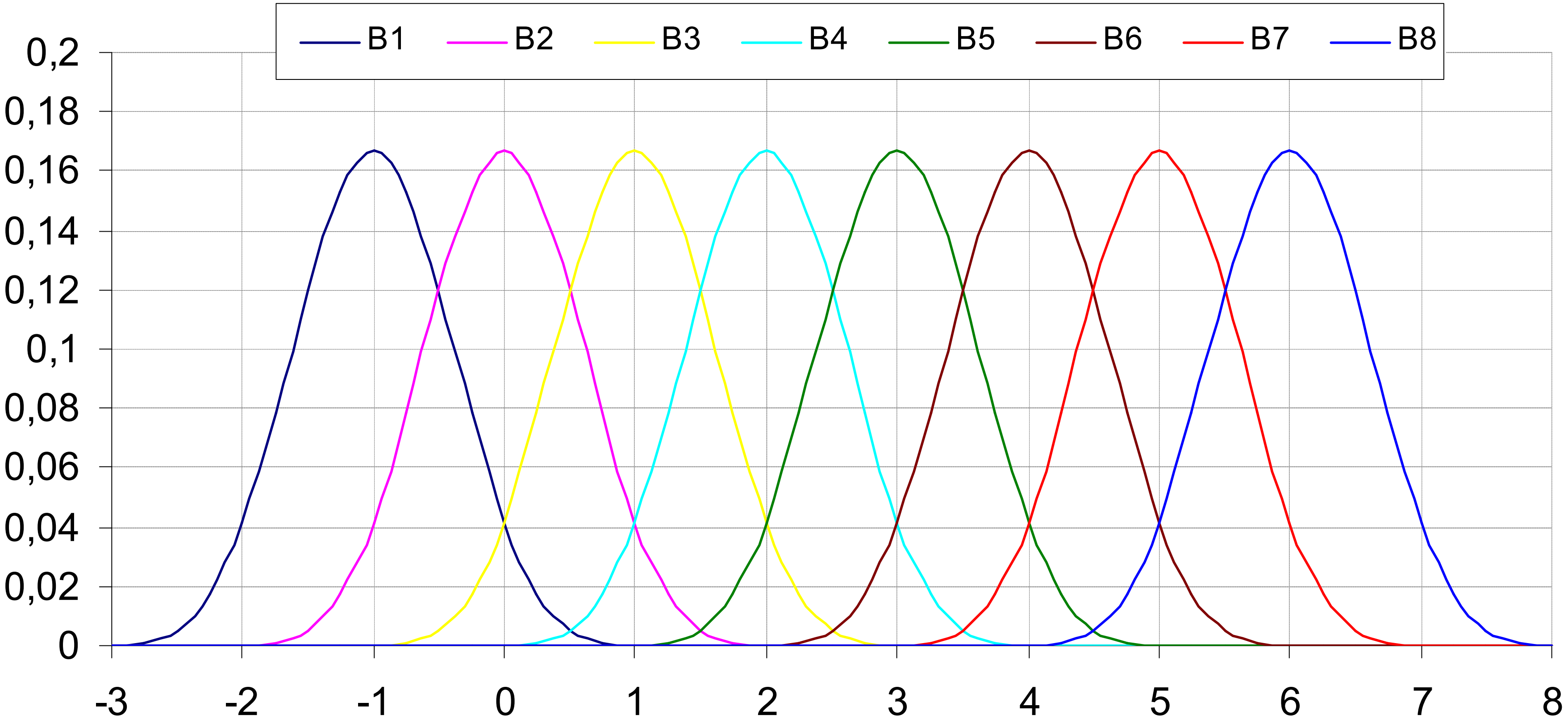
B-spline different orders

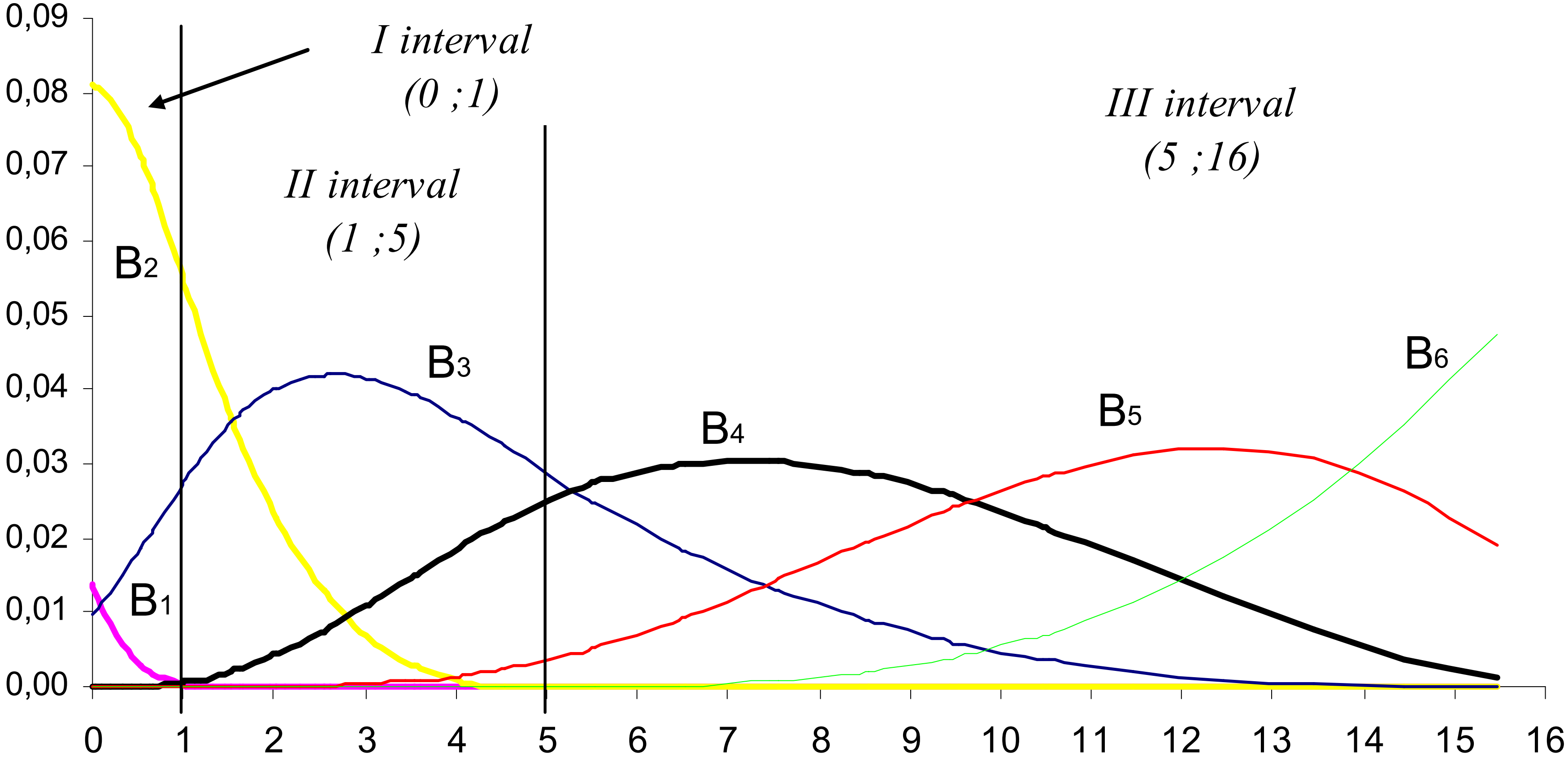


B-spline of 3rd orders



B-spline base

B-spline base example



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Yield construction

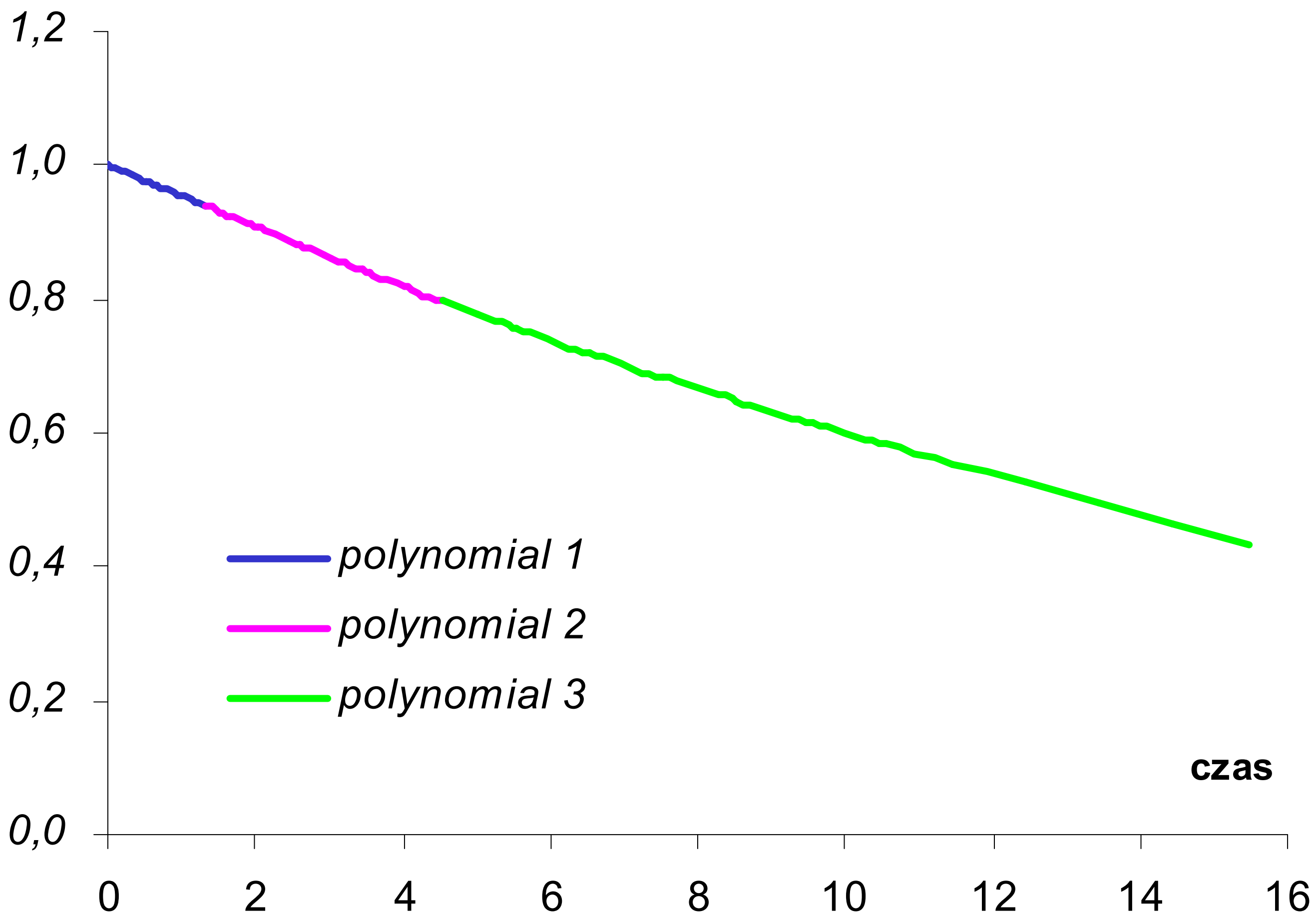
Finding such parameters $\gamma_{1}, \gamma_{2}, ,\ldots, \gamma_{n+3},$ *that*:

$$\frac{\sum_{i=1}^{k}(P_i - \overline{P_i})^2}{k} \to \min$$

$$\begin{bmatrix} \overline{P_1} \\ \overline{P_2} \\ \ldots \\ \overline{P_k} \end{bmatrix} = C \cdot \begin{bmatrix} \overline{\delta}(\tau_1) & \overline{\delta}(\tau_2) & \ldots & \overline{\delta}(\tau_m) \end{bmatrix}^T = C \cdot B^T \begin{bmatrix} \gamma_1 & \gamma_2 & \ldots & \gamma_{n+3} \end{bmatrix}$$

$$i(\tau_l) = -\frac{\ln \delta(\tau_l)}{\tau_l}$$

Discount Yield



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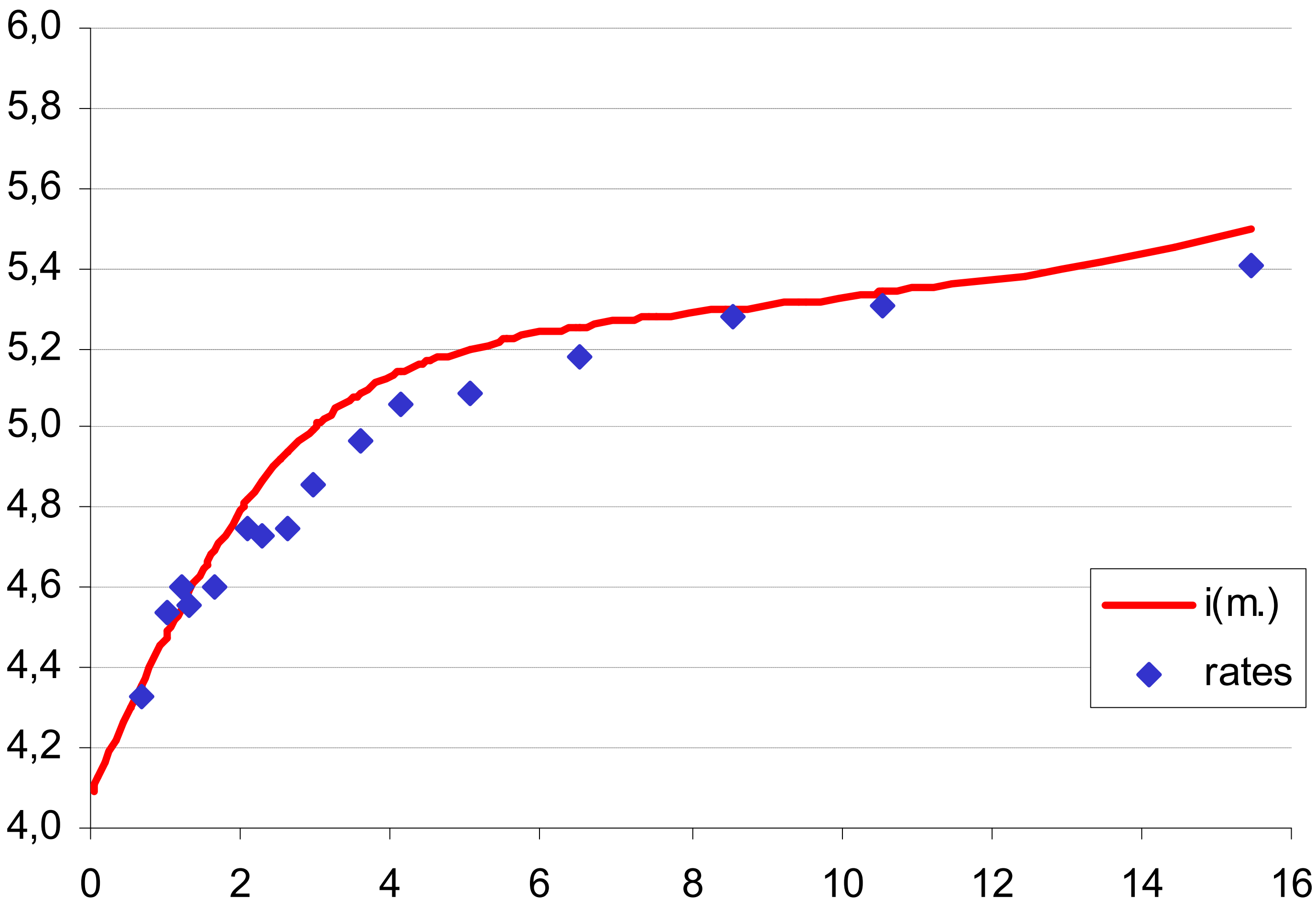
Zero coupon yield (based on splines)



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Comparison of models

	N-S	SV	Cubic
smoothness	+	+	+
flexibility	-	+	+
stability	+	+/-	+

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